

1975

An experimental study of two approaches to teaching high school geometry

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An experimental study of
two approaches to teaching high school geometry

by

Walter Herbert Wood

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Department: Professional Studies
Major: Education (Higher Education)

Approved:

Signature was redacted for privacy.

In Charge of Major Work

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For the Major Department

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For the Graduate College

Iowa State University
Ames, Iowa

1975

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ACKNOWLEDGMENTS

The investigator wishes to express appreciation to the many people who contributed to the completion of this study.

Sincere appreciation is expressed to Dr. Ray J. Bryan, Dr. Milton D. Brown, Dr. Trevor G. Howe, Dr. Orlando C. Kreider, and Dr. Clarence H. Lindahl for serving as members of my committee and providing encouragement and assistance. I would like to especially thank Dr. Bryan for chairing the committee and for helpful suggestions during the writing of this dissertation and Dr. Howe for helpful statistical advice. Thanks also is expressed to Dr. Rex Thomas for his assistance as the data were prepared for the computer and appropriate programs were deciphered and executed.

I am also grateful for the cooperation of Dr. Ralph Farrar, principal, and the mathematics department staff at Ames Senior High School where this study was conducted. A special thanks is given to Phil Johnson who participated as an instructor and offered helpful suggestions throughout the experiment.

To my wife, Georgia, and daughters, Sheryl, Glenda, and Stephanie, I will always be grateful for expressions of encouragement, patience, and faith during my years of graduate work.

CHAPTER I. INTRODUCTION

The role of geometry in the secondary school has been a controversial issue debated by mathematicians and educators for many years. What should be the nature or characteristics of high school geometry? Does it need to be a full year course? Should geometry be integrated with other high school mathematics courses? Should geometry be the chief vehicle for teaching the deductive method of reasoning? Is the course content appropriate in light of new mathematical developments? How much emphasis should be placed on the abstract nature of geometry? Why do so many students express negative feelings toward their study of geometry? These and other questions have been debated in educational circles since the turn of the century. This experiment was designed to study the last four of these questions as they relate to the importance of mathematical structure to the study of geometry and its effect on student attitudes toward high school mathematics.

Need for the Study

Curriculum changes in mathematics have been numerous in recent years. Especially during the last ten to fifteen years the changes of the "new math" have gained much attention as they have been implemented in both the elementary

and secondary schools. Prominent educators have cooperated through curricular writing groups such as Science Research Associates (SRA), School Mathematics Study Group (SMSG), the University of Illinois Committee on School Mathematics (UICSM) and others to provide leadership, textbooks and test materials for these changes.

Although there seems to be some measure of agreement among educators relative to the "new math" changes that have taken place in arithmetic and algebra this is not the case for high school geometry (4). Because of the lack of agreement about how geometry should be changed there actually has been very little change in the way it is being taught in most schools. Seymour Schuster (26) expressed the view of many when he discussed the status of geometry at the International Conference on the Teaching of Geometry held in Carbondale, Illinois, March, 1970:

The secondary schools of the U.S.A. now have--and have had for most of the twentieth century--Euclidean Geometry as the content of their geometry curriculum. ... The primary focus of the teaching has been on constructing the formal systems: the postulates, definitions and formal proofs. The important thing to note is that this focus has not changed by the reform of recent years. The reform assumed the existing philosophy, namely that tenth-year geometry was to be devoted to an axiomatic development of Euclidean Geometry, and exerted its energies to preserve Euclidean Geometry as a model of logical thinking. To be sure, some of the new books have enriched the content of the course in other ways,.... Thus, the reformers did not change the basic aim of the high school course nor were successful in altering the spirit in which the geometry was to be studied.

There are several new tenth-grade courses in geometry that are being tried experimentally in schools throughout the nation but none has received wide acceptance. Geometry continues in most high schools as a full year course in a modified version of Euclidean geometry with primary emphasis on the concept of formal proof based on an axiomatic structure. The content is developed as a unified, abstract mathematical system based on undefined terms and certain reasonable assumptions (axioms) from which the remaining information is obtained by proper use of deductive reasoning (proof).

The effectiveness of this formal approach is questionable for a large percentage of high school students. Numerous educators in the field of mathematics have expressed dissatisfaction with this approach to geometry. Charles Brumfiel (9, p. 99) has stated:

Critics of present practice argue that a concern with axiomatics, logic, and formal proof slows the development of geometry and actually has only superficial impact on students. It would be illuminating to run a series of studies and find out what understanding of the structure of geometry students retain at three stages in their education: (a) at the end of their geometry course, (b) at high school graduation, and (c) some years after their departure from high school.

Brumfiel continues to relate how in 1954 he started gathering information from his university students on their understanding of the axiomatic structure of high school geometry. He concludes from his inquiries that (9, p. 102):

Students of 1954 who studied an old-fashioned hodge-podge geometry had no conception of geometric structure. Students of today [1971] who have studied a tight axiomatic treatment also have no conception of geometric structure.

To avoid misunderstandings from the above quotations it should be said that Brumfiel (9, pp. 102-3) does not advocate an abandonment of the teaching of axiomatic structure in geometry but rather suggests that attention to the axiomatic structure be concentrated near the end of the course.

This study relates to the question of emphasis to be placed on the formal mathematical structure of geometry. The axiomatic approach to nearly all areas of mathematics has been the key issue in the mathematical developments of recent years. Most algebra text materials are now written to illustrate the axiom-definition-theorem development of this subject. The spirit of this approach can be seen in junior high mathematics where the properties of the real number system are studied and on down into lower elementary levels with attempts to create better understanding of the basic mathematical concepts normally taught there.

These developments and other viewpoints about modern mathematical needs are prompting many suggestions for change in geometric instruction. The added emphasis on structure at all levels of instruction certainly should have an effect on the teaching of geometry which in the past has been about the only course where structure has been studied in detail.

Howard Fehr (12, p. 370) discusses this point in the 36th yearbook of the National Council of Teachers of Mathematics which is devoted entirely to the teaching of geometry:

An important reason for the survival of Euclid's geometry rested on the assumption that it was the only subject available at the secondary school level that introduced the student to an axiomatic development of mathematics. This was indeed true a century ago; however, recent advances in algebra, probability theory, and analysis have made it possible to consider using these topics, in an elementary manner, to introduce axiomatic structure in the secondary school.

Irving Adler (1, p. 229) expresses a similar view:

As a result of the changes in the mathematics curriculum during the last decade, the students have experience with deductive proofs in ninth-grade algebra, and even in seventh- and eighth-grade arithmetic. Consequently, deductive reasoning doesn't need all the attention that we now give it in the tenth grade.

When one views the number of mathematicians and educators currently expressing dissatisfaction with high school geometry it seems certain that experiments with new approaches to this problem will be on the increase. Judging by educational practice of the past, many (if not most) of these experiments will not be organized so that variables can be controlled and the educational effectiveness measured. Research provides a more objective, systematic method for curriculum changes which is far superior to the reliance on personal feelings of teachers and other authorities.

Statement of the Problem

This study has been designed to investigate the effectiveness of the instructional emphasis and content of the tenth grade geometry course at Ames Senior High School, Ames, Iowa. This course as it has been taught for several years, is the traditional full-year of Euclidean geometry developed through the definition-axiom-theorem, formal-structure approach. The text, Modern School Mathematics Geometry, 1969 edition, by Jurgensen, Donnelly, and Dolciani has been used for the past four years and will be used by the control group in this study.

Geometry teachers and others working closely with the students have expressed doubts about the effectiveness of this traditional approach. The attitudes of students toward their high school education, trends in post-high school educational plans, and changes in the tenth-graders' mathematical background as they enter Ames Senior High are some of the factors which have been mentioned as contributors to the problem. More specifically, the task of developing the traditional geometric content as a single abstract mathematical system is a difficult one and often not appreciated nor understood by students. It is felt that other worthwhile objectives for the geometry course could be achieved with a less formal instructional approach.

Text materials for the experimental group were prepared

by Phil Johnson and Walter Wood, the two instructors for the classes used in this study. The title of the text, "Geometry, An Informal Approach," indicates the change in instructional emphasis in the experimental group. Formal, two-column proofs are not included in this text. This text includes the same topics found in the text for the control group except those which directly relate to the development of and practice with formal proofs. More emphasis is placed on the application of the geometric properties in the experimental text.

As the brief description of text materials indicate, this study compares two instructional approaches to the teaching of tenth grade geometry. The effectiveness of the two approaches is measured in two categories, geometry achievement and student attitudes. The objectives which follow will add further details.

Objectives of the Study

The objectives of this study are: (1) to determine the effect on achievement in traditional high school geometry topics when the mode of instruction is less formal and students are not required to make formal proofs of theorems, (2) to determine if the less-formal instructional approach coupled with simulated career applications of geometric properties will significantly improve the understanding of

those geometric properties when compared with the traditional, formal-proof instructional approach, (3) to determine if there is a significant relationship between instructional approach and sex, I.Q. level, or previous mathematical achievement of the student, (4) to compare attitudes of students in the two groups toward mathematics in general and toward their high school geometry course, and (5) to determine what effect the absence of formal proof and lack of emphasis on axiomatic structure in the informal instructional approach might have on the students in the experimental group when a coordinate geometry unit is studied which includes proofs.

Hypotheses to be Tested

1. There is no significant difference in group means of students who have studied geometry under an informal instructional approach with career-oriented units (experimental) and those who have studied under an axiomatic, formal-proof approach (control) when tested on achievement in traditional geometric properties by teacher-made unit examinations.

2. There is no significant difference between mean scores of the experimental and control groups when given a standardized, full-year geometry achievement test.

3. There is no significant difference in performance

of the experimental and control groups, as measured by a teacher-made achievement test, when a coordinate geometry unit is studied that includes coordinate geometry proofs.

4. There is no significant difference between mean scores of the experimental and control groups when given an attitude-toward-mathematics scale.

5. There is no significant difference between mean scores of the experimental and control groups when given an attitude-toward-geometry scale.

6. There is no significant difference in the relationship of achievement and IQ scores for the experimental and control groups.

7. There is no significant difference in the relationship of geometry achievement scores and grades seven through nine mathematics grade-point averages for experimental and control groups.

8. There is no significant difference in the relationship of scores from attitude scales and IQ scores for the experimental and control groups.

9. There is no significant difference in the relationship of scores from attitude scales and grades seven through nine mathematics grade-point averages for the experimental and control groups.

10. When achievement is used as the criterion variable, there is no significant interaction between instruc-

tional method and sex of the student.

11. When attitude is used as the criterion variable, there is no significant interaction between instructional method and sex of the student.

Source of Data

Ames Senior High School is located in the midwestern city of Ames, Iowa, with a population of approximately 44,000, and home of Iowa State University which contributes an additional student population of approximately 20,000. Also located in the city is the Iowa State Highway Commission headquarters, an Atomic Energy Commission laboratory, and a National Animal Disease Laboratory; all have college graduates for a large percentage of their employees. This in turn has an effect on the needs of the student population and the instructional program at the senior high school. A recent survey of the student body indicated that fifty-one percent of the students' fathers and seventeen percent of their mothers are engaged in professional or semiprofessional occupations (6). The 1970 Ames census indicated that the median number of years of school completed by adults in the city, 25 years of age and over, was 16.4 for males and 13.5 for females (29, p. 368). These facts help to explain the favorable educational climate of the community.

Ames Senior High School has approximately 1,250 students

in grades ten, eleven and twelve. There were 440 students in the tenth grade, the class providing the majority of the subjects for this study. At the junior high school level ninety-five of the tenth graders participated in an accelerated mathematics program, taking algebra in the eighth grade and geometry in the ninth grade. Hence, many of the higher ability mathematics students have not been included in this investigation. All participants had the first year of algebra in their mathematics sequence. The students who had preregistered for geometry were first randomly assigned to experimental and control groups, then scheduled into four experimental and five control class sections. The nine geometry sections included 196 tenth graders with an average I.Q. of 114 and 23 eleventh graders with an average I.Q. of 106.

Delimitations of the Study

The conclusions that may be drawn from this investigation are limited by the various factors which contribute to the educational setting in which it was conducted. One major factor is determined by the characteristics of the participating students. Most were tenth graders who for one reason or another did not choose to accelerate in their mathematics sequence at the eighth grade level. This means that many of the higher ability mathematics students were

not included in this study. Even so, this sample of students may be representative of many geometry classes since schools often have acceleration programs starting at the eighth grade level.

Another limiting factor is the text materials used. The results are applicable only for the geometry topics covered in this experiment. The text used by the experimental group was prepared by the participating instructors, Phil Johnson and Walter Wood, and is not commercially available. Although every attempt was made to provide unbiased instruction, the findings of this study may be applicable only to students attending this high school and taught by the two instructors who participated in this investigation.

CHAPTER II. REVIEW OF THE LITERATURE

Literature reveals that in recent years an increasing number of educators and others who use mathematics are concerned about secondary school mathematics programs. The "new math" reform of the last fifteen years has brought changes at all levels of mathematics education. The course least affected by these changes, however, has been high school geometry.

Quast (22) made an historical study of geometry in secondary schools of the United States from 1890 to 1966. Viewing the present status of the high school mathematics curriculum from an historical perspective is helpful when considering possible alternatives for the future. One interesting fact brought out by Quast is that many of the changes which have taken place or have been advocated during the last few years had been suggested, and in some cases attempted, at some previous time.

In his survey Quast reviews the efforts of many organizations, committees and individual educators that have attempted to influence the secondary mathematics curriculum since 1890. The extent of concern about geometry instruction during this time is indicated in the following statement by Quast (22, pp. 7, 12):

Beginning with the Committee of Ten in 1892, many organizations have attempted to analyze, clarify, and

influence the aims, content, and methodology of secondary school geometry.... Perhaps no subject in the secondary curriculum has been subjected to more criticism through the years. Moreover, since 1892 there has been an almost constant effort to change either the content or the classroom procedures for geometry.

Although suggestions for revision of high school geometry have been numerous since the turn of the century real changes have been minimal and slow in coming. One of the changes recommended by committees studying the curriculum in the early nineteen-hundreds was that geometry instruction should be made less formal and more practical. Quast describes the response in the geometry classroom by saying (22, p. 98):

The period from 1890 to 1920 saw great changes in secondary education, but few real modifications in the teaching of geometry. Even the advent of mass education failed to bring general adjustment from the traditional, formal approach to the content of demonstrative geometry.

The period from 1920 to 1945 included World War II and the growth of the progressive education movement which exerted pressure on all curriculum areas. Again the actual changes in the classroom were minimal even though numerous suggestions for improvement were made.

The first decade after the end of World War II saw a continuation of past practices in mathematics teaching, but the period after 1955 has been described as a "mathematics revolution" because of widespread curriculum revision in the direction of what has become known as the "new" or "modern"

mathematics. One common characteristic of the changes that have occurred at the various levels of mathematics is an increased emphasis on structure. This means that the tendency has been for other mathematics courses, especially at the secondary level, to become more like geometry in organization of content. Although changes have also occurred in high school geometry during this period, they have been less significant than at other levels. Quast made the following statement about this period in time (22, p. 225):

The changes which occurred in mathematics education from 1955 to 1965 were without precedent in the history of education, for never had schools so rapidly implemented recommended courses and modified existing programs. The development of numerous new approaches to mathematics education resulted in widespread acceptance of new ideas. Unfortunately, the mathematics course least affected by the reform movement was geometry. Although changes did occur in the teaching of geometry traditional ideas and procedures persisted.

An important question to consider is why curriculum changes were so numerous during this period and previous recommendations had received little response. One influencing factor was the number of curriculum study groups functioning at one time, providing a variety of alternatives for mathematics teachers. Prior to this only one major reform group would be making recommendations during a given period in time. Another distinguishing characteristic of this reform movement has been that curriculum study groups have also been curriculum writing groups, preparing materials

to be used in the classroom and making it much easier for their proposed changes to be accomplished. Teacher training has also been a contributing factor. The financial support provided by the National Science Foundation during this period, for training, caused teachers to be more receptive to the idea of change and better prepared to respond to new materials.

Availability of instructional materials is an important factor in any attempt to change a course of study. The National Longitudinal Study of Mathematics Abilities (NLSMA), organized to assess the outcomes of various new mathematics programs, included data related to the effectiveness of text materials prepared by several different publishers and writing groups. While reviewing the results of this study Begel (7) pointed out that the textbook does have a strong influence on what students learn. The data revealed that, on the average, if a topic is included in a text then students will learn it and if the topic is not in the text then students do not learn it. In other ways, however, the text is not as strong a variable as one might think. Begel (7, p. 210) states:

Other textbook variables seem to be less important. For example, we found that the style in which a textbook is written is not very important. We cannot find any evidence that careful editorial polishing of a manuscript has much effect on student learning. However, in a slightly different direction, we did find that among the texts that seemed to be less effective than

others were most of those that I would classify as being overly formal. Again, however, the differences were not large.

The literature reviewed in the remainder of this chapter is grouped into three categories: (1) Related Mathematical Structure Research, (2) Related Instructional Approach Research, and (3) Related Mathematical Attitude Research.

Related Mathematical Structure Research

The proper balance between the theoretical and the practical has been discussed by mathematicians and educators for decades. These discussions have often focused on the high school geometry course. As one would expect, opinions vary greatly about the importance of the theoretical structure of geometry to high school students. Irving Adler (1) expressed the view that logical reasoning no longer needs so much attention at the tenth-grade level because of its increased emphasis at other levels. At the same time, the following statement (1, p. 229), reveals the importance Adler places on the deductive development of geometry:

There are many good reasons for stressing axiomatic-deductive reasoning in tenth-grade geometry. First, it is an important type of reasoning that all people should learn to appreciate. Secondly, tenth-grade students are ready for this type of reasoning, because (a) it has been completely axiomatized, (b) it is intrinsically interesting because it relates directly to the students' intuitive experience with physical space, and (c) it is not entangled with the biases and emotions that impede objective thinking in other subjects.

Opinions at the other extreme and of varying degrees in between have been expressed by others. Seymour Schuster (26, p. 77) argued that, "focusing on the formal structure of [high school] geometry is a serious mistake which has had some unfortunate results." Rev. D. B. Smith (27, p. 87) stated that, "The proper moment for a really close scrutiny of the foundations of this edifice [formal development of geometry] is at the university, when greater maturity has been achieved."

Myers (17) in his doctoral thesis, analyzed some of the issues related to the role of the axiomatic method in secondary school mathematics, focusing on what he termed the "curriculum revolution of the past decade." One issue included is the argument that learning the axiomatic method will help students become more critical in their thinking outside the mathematical context. Myers reviewed several empirical studies and from these concluded that transfer is unlikely to occur unless the method is taught with that goal in mind; this normally is not the case in high school geometry.

Another argument by proponents of the axiomatic method is based on the assumption that mathematics and axiomatics are essentially equivalent, indicating that a student is not studying mathematics unless the axiomatic method is used. Myers quoted a number of mathematicians on both sides

of this issue and noted that "there is no compelling reason to believe that a knowledge of axiomatics is a prerequisite for those whose aim is to apply the content and methods of mathematics in other fields."

A third issue considered by Myers was that students will master more thoroughly the content of mathematics when it is taught by the axiomatic method. The conclusion was that although additional research is needed on this issue, available evidence seems to indicate that use of the axiomatic method has no effect on a student's comprehension, retention and ability to apply the mathematics he learns at a later time.

Another controversial issue investigated by Myers has to do with motivation. Proponents of the axiomatic method suggest that the logical structure of the system should be appreciated by high school students and also capture their interest. Some who are critical of this viewpoint say that motivation for the study of mathematics comes from its use as a tool for solving real-world problems. Myers concludes that (17, p. 328):

... whether or not students are interested in an abstract, axiomatic presentation of mathematics is an empirical issue which has not yet been studied. But whether or not students should be interested in axiomatic mathematics is a philosophical question, the answer to which depends upon one's view of the nature of the mathematical enterprise.

In recent years there have been a number of studies

conducted to compare the so-called "modern" and "traditional" mathematics programs. Modern usually refers to the axiomatic approach to a subject; beginning with a set of undefined terms and a set of assumed properties (axioms), making definitions of new terms as needed and deducing new properties (theorems) by methods of logical deduction (8, p. 498). If the grade level is too low for a complete axiomatic development then concepts and terms are presented and reasoning patterns used that are consistent with the approach. A large percentage of the studies assessing the modern approach have not been in the area of geometry since changes have been more numerous at other levels. Historically the approach to geometry has been axiomatic with the reform of recent years bringing a more refined and rigorous treatment.

Yasui (35) compared achievement and attitudes of students enrolled in modern and traditional programs after they had been in the program for three years. The experimental group consisted of 141 grade twelve modern mathematics students who had started in the program in grade ten. The control group consisted of 125 students selected from high schools which were not exposed to a modern program.

Some of the conclusions made by Yasui after reviewing the literature are as follows (35, pp. 48-50):

1. An investigation of algebraic achievement as measured by various standardized tests shows that the modern

mathematics program is just as effective or better than the traditional mathematics program.

2. ... it would appear that the modern mathematics program is just as effective or better than the traditional program in developing 'mathematical reasoning' competence.

3. In evaluating arithmetic fundamentals and manipulative skills between the modern and traditional programs the results were inconclusive.

4. ... two studies reviewed indicated that the modern mathematics program is partial to students in the upper intelligence level.

5. There appears to be an agreement among most studies that there exists a positive relationship between mathematical achievement and attitude.

In this study Yasui used the Contemporary Mathematics Test, developed by the California Test Bureau, to measure achievement and the Mathematics Inventory developed by Cyril J. Hoyt and Donald G. MacEachern to measure attitudes. The quantitative scores of the SCAT were used to control individual differences in ability. Making use of common test items, representative of both the modern and traditional programs, Yasui concluded that the modern mathematics program of Edmonton, Alberta, Canada developed greater understanding of mathematics in high school students than the traditional program. There was no significant difference in attitudes of students between the two programs.

After reviewing studies related to the modern versus traditional approach and reading comments by educators and mathematicians on the subject there does not seem to be

conclusive research evidence favoring either approach. There are many variables that play an important role in the decision of which approach is best for a particular group of students. There seems to be some evidence that for many students the modern approach produces a better understanding of mathematical concepts while the traditional approach possibly provides better training for the basic mathematical skills. And as pointed out in Yasui's study there is some evidence that the modern approach is partial to the upper intelligence level.

Related Instructional Approach Research

Research has been done on a variety of approaches to high school geometry; analytic, transformation, and vector approaches are some of the alternatives to geometric content that have received considerable attention in recent years. Different methods for presentation of content have also been studied, such as large-group small-group instruction or independent study. The research being reviewed in this section is related to what has become known as "formal" and "informal" instructional approaches. The formal approach to geometry instruction usually refers to an axiomatic development of the content which includes emphases on mathematical structure, deductive logic, and formal proofs. The informal approach usually includes less emphasis on mathematical

structure and the formal proof and increased use of inductive reasoning.

Kellogg (15) conducted an investigation at the University of Minnesota High School to assess the importance of deductive reasoning, inductive reasoning, and geometric applications. Included in the study were fifty-five sophomores randomly assigned to three geometry classes. One class received practical geometric applications along with deductive and inductive experiences. A second class received instruction stressing inductive methods along with some deductive experience but no practical applications. Deductive methods were emphasized in the third class with both applications and inductive experiences being excluded. The basic geometric content was held constant in the three classes.

Criterion measures for the study were achievement, retention and attitude. The analysis of variance and covariance were used to make comparisons among classes. Differences among classes were not significant for overall achievement as measured by the Cooperative Plane Geometry Test and not significant for retention or attitude as measured by tests prepared by the investigator. In comparisons of various subtopics the deductive class was superior to the inductive class in two instances. These significant differences were noted on tests containing questions emphasizing

deductive reasoning. The application class was superior to the inductive class in one case.

In summarizing his study Kellogg stated that the "experiment does not provide definitive conclusions as to the 'best' method of teaching plane geometry." It did provide evidence that students in that particular experimental situation who studied by deductive methods performed better on tests emphasizing those methods and just as well on the other criterion variables measured.

Nichols (18) also conducted an experimental investigation of the relative effectiveness of deductive and inductive instruction. He titled the two methods the "dependence approach", which would fall in the category of what is generally considered the deductive method, and the "structured search approach", falling in the category of inductive methods. In the dependence approach class students "depended on the teacher for statement of assumptions, theorems, definitions, and verbalization of principles." The students in the structured search approach "discovered every relationship ... through a series of concrete experiences with drawings of geometric figures and through mensuration." The criterion test measured success by the students' knowledge of geometric vocabulary, critical thinking ability, ability to solve problems and the ability to use a ruler and protractor in mensuration problems.

The data collected in this experiment led Nichols to conclude that the Dependence Approach (deductive) and the Structured Search Approach (inductive) were equally effective in teaching plane geometry to high school freshmen.

Both deductive and inductive reasoning are important in the study of mathematics. The question often debated is which kind of reasoning should receive the greater emphasis and at what level should one or the other receive more attention. The deductive side of mathematics has traditionally been concentrated in the tenth grade geometry course. Not only is deductive reasoning practiced here but the content has customarily been organized into a single deductive system. The formal proof is an important part of the development of this deductive system. It has been suggested that geometry need not be the major source for training in formal proofs and deductive reasoning but that algebra can now provide better opportunities for this (25, p. 402).

Deductive proofs are an essential part of the axiomatic development of geometry. The form that should be followed when constructing a proof is a debatable question however. VanAkin (30) conducted an experiment to determine the effect of structure in proof upon the following two objectives for high school geometry (30, p. 9):

1. To develop a knowledge of geometric facts and relations;
2. To develop the ability to reason logically.

His study was conducted during the 1970-71 school year using two high school geometry classes in each of two separate midwestern high schools; one serving an urban-suburban-rural community and the other serving an inner-city-like community. Text materials were written by the investigator so they would differ "only in the presentation of book proofs; one treatment group to be presented with a two-column proof for a theorem, and the other to be given a paragraph proof for the same theorem" (30, p. 50). The data collected revealed no significant advantage for either method of proof with respect to the knowledge of geometric facts and relations variable or the logical reasoning variable. VanAkin states that his study suggests (30, p. 251):

....a teacher of high school geometry does have at least some freedom of choice relative to the structuring of deductive proofs, without putting either of the two time-honored objectives for the course in serious jeopardy.

Another study related to the question of formality and informality of presentation in geometry instruction was conducted by Bassler, Curry, Hall, and Mealy (6). Subjects for this investigation were seventh graders, categorized as middle ability students (mean I.Q. score of 100) at the Apollo Junior High School of Nashville, Tennessee. Seventy-two students were assigned at random to one of six cells representing a single combination of two instructional variables. The variables investigated were presentation mode, formal

deductive versus informal inductive, and number of exercises completed for each topic studied. A two-by-three factorial analysis of variance design was used with the two modes of presentation forming the first dimension and number of practice exercises completed forming the second dimension.

Programmed learning materials were used for all groups covering the nonmetric geometry topics "Specifying Sets, and Separations of Sets Using Points, Lines, and Curves." Definitions and postulates were stated as such in the formal approach but not in the informal approach. Generalizations of properties preceded illustrative examples in the formal approach while examples preceded the generalizations in the informal approach. Third person language was used in the formal approach while a more conversational language was used in the informal.

Criterion instruments were constructed by the investigators to assess both immediate effects and also retention after six weeks. The statistical analysis revealed that neither the formal nor the informal approach resulted in consistently higher mean scores on any of the measures and the number of exercises variable had no significant effect. Generalizations from this study are limited since programmed instruction was used and a limited number of topics were covered, and those topics were definitional in nature.

Another factor to be considered along with an instruc-

tional approach is the time that should be devoted to the practical aspects of the subject. Applications can be a part of the instructional method or a means to show the student the usefulness of what is being studied. Schuster (26, p. 78) has said that, "The overemphasis on axiomatics has resulted in an underemphasis on applications." Zoll (36, p. 1) suggested that, "... if the usefulness of the subject in everyday affairs were made known to the learner, the learner would either learn more about the subject or desire to learn more about it."

Zoll (36), in his doctoral dissertation, investigated the relative merits of teaching plane geometry with varying amounts of applications. Six classes of plane geometry at the Levittown Memorial High School in Levittown, New York participated in his study. The geometric content was the same for three experimental and three control classes. The instructional approaches differed in the type of homework problems assigned. Control classes worked problems from the textbook while the experimental classes were assigned varying amounts of application problems in addition to some textbook problems.

In this full-year course in plane geometry tests were administered at the end of each major unit. The tests were designed to measure knowledge of geometric facts, the ability to solve original geometric problems, and ability to apply

geometric facts in practical problems. In addition pre- and post- tests were used to measure growth in algebraic and arithmetic competencies. Data collected were analyzed to investigate a total of six sub-problems and no significant differences were found to be related to the varying amounts of applications. If practice in applications does have an effect on the abilities measured in this study then the approaches used in this study were not different enough to produce significant results.

Related Mathematical Attitude Research

Much has been written about the relationship of attitudes to learning. Writers agree that favorable attitudes are important to learning but many admit that it is not clearly understood just what role attitudes play in the learning process. After reviewing what many have found through research about the relationship of attitude to achievement in mathematics Aiken (2, p. 559) made the following statement:

The relationship of attitudes, which are integrally related to expectations, to performance appears to be especially important in mathematics. One study (Brown and Abell, 1965) clearly demonstrated that the correlation between pupil attitude toward a subject and achievement in that subject was higher for arithmetic than for spelling, reading, or language. ... these studies are not always consistent in their findings, although they generally report low to moderate correlations between the [attitude and achievement] variables.

Some have felt that improved attitudes would accompany the "modern" mathematics curriculum reforms with their increased emphasis on understanding of concepts and mathematical structure. Research evidence generally has not favored either approach, modern or traditional. There are still advocates of both sides of the issue, but it is the opinion of this writer that evidence is pointing toward a less favorable attitude toward the modern curriculum by the average or below average student and either no significant difference or slightly more favorable attitude by the above average student.

The School Mathematics Study Group (SMSG) has contributed more to this mathematical reform era than any other study group. A five year study, the National Longitudinal Study of Mathematical Abilities (NLSMA), was conducted by the SMSG Staff to assess the effectiveness of SMSG materials. After reviewing the data produced by this study, Dr. Edward G. Begle, Director of SMSG, made the following comment about student attitudes (7, p. 212):

We can, however, report that student attitudes towards mathematics seem to be rather favorable at the beginning of fourth grade and improve slightly during the remainder of elementary school. However, at the beginning of junior high school, student attitudes toward mathematics begin a slow but steady drop that continues to the end of high school. These attitudes seem not to be affected by the nature of the curriculum to which the student has been exposed.

The "nature of the curriculum" mentioned in this statement

includes both traditional and modern text materials being used across the nation during the period 1962-67.

Numerous studies have compared SMSG and traditional materials since 1958 when this group was organized. Phelps (20) conducted a study with fifth and eighth grade students and found no significant difference in attitudes toward SMSG or traditional text materials at the eighth grade level but found a significant favorable attitude toward the SMSG materials at the fifth grade level. Woodall (34) compared attitudes and achievement of pupils taught by SMSG and traditional materials at the fourth, sixth and eighth grade levels over the six year period 1959-65. The study involved over one-thousand students at each level and it was found that SMSG materials did not produce significant positive attitudes or achievement at any of these levels.

Begel (7, p. 212) in the quotation given above mentions that the NLSMA study shows attitudes toward mathematics becoming progressively less favorable through the junior high and senior high school years. Anttonen (5) investigated this question of stability of mathematics attitude from the elementary to the secondary level. He conducted a six-year longitudinal study starting with 607 fifth and sixth grade students and found that mathematics attitude scores obtained for the same subjects at the late elementary and late secondary school levels did not correlate very highly. He also

found low relationship between elementary attitude scores and mathematics achievement at both elementary and secondary levels. He found a somewhat higher relationship of secondary attitude scores and secondary mathematics achievement. This seems to indicate that attitudes fluctuate considerably at the elementary level and begin to become more crystallized at the secondary level. This study and the NLSMA study seem to infer that the tenth grade geometry course comes at an important time in the student's mathematical life; a time when attitudes toward mathematics are fluctuating yet beginning to crystallize for many students.

What are the factors which contribute to favorable or unfavorable reactions toward a subject? Poffenberger and Norton (21) investigated this question in a study involving all incoming freshmen at the University of California of Davis, California. A questionnaire was given these students in the fall of 1955, measuring their attitudes toward various subject matter areas and toward school in general. Two groups were selected from the total population; a "positive" group who had indicated a strong liking for mathematics, and a "negative" group, those who had indicated a strong dislike for mathematics. These two groups did not differ significantly in their attitudes toward school in general nor in their over-all high school grades. The two groups were then compared by their responses to questions

designed to determine factors which might have contributed to their strong feelings about mathematics. The following statements were included in the summary of the data analysis (21, p. 175):

The factors that significantly differentiated the two groups were the attitudes of the fathers toward mathematics and the expectations of both fathers and mothers of mathematical achievement on the part of their children ... The same was true in the case of parental encouragement in mathematics courses. The two groups also differed significantly in their attitude toward their course in algebra one. ... The research findings in this paper give evidence that the present lack of interest in mathematics is largely a cultural phenomenon pervading not only the educational system of the country but also the family as an institution that conditions the attitudes of children.

Roberts (24) investigated the attitudes of high school students toward mathematics and included the teacher attitude variable in his study. Roberts (24, p. 785) commented that, "If, as is currently accepted, attitudes are transmitted as part of our learned heritage, then teachers play a part in imparting attitudes." Opinions about the effects of teacher attitudes in the mathematics classroom vary from the belief that students reflect their teacher's attitudes to the view that student attitudes are primarily a result of the home environment. Roberts sampled one junior high school and two senior high schools in the state of New Jersey. A questionnaire was administered to 323 eighth, ninth and twelfth graders and to 112 faculty members in these schools. The questionnaire included scales to measure attitudes toward

mathematics as a process, attitudes about difficulties of learning mathematics and attitudes toward the place of mathematics in society.

When means were compared for the students as a whole and the faculty as a whole on the combined scale scores, no significant difference was found. Some differences did appear on the individual subscales. For the scale "attitudes towards mathematics as a process", mean scores were significantly higher for the teachers and for the scale "attitudes toward the place of mathematics in society", mean scores for the students were significantly higher. Roberts concluded from the study that the small differences between student scores and teacher scores was an indication that once attitudes toward mathematics are adopted, they remain relatively stable over the years.

Mathematics teachers often come in contact with students who perform inadequately in mathematics while their performance in other subjects is average or above average. One possible explanation of this problem was investigated by Degnan (11) in a study designed to compare the attitudes toward mathematics and general anxiety level of a group of junior high school students whose achievement in mathematics was average or above with another group of students of equivalent general ability whose achievement in mathematics was below average. One group of twenty-two students classified

as "achievers" in mathematics and the other group of twenty-two classified as "underachievers" in mathematics were equated on sex, age, ethnic background, general school attitude, health, reading grade level and academic average (in non-mathematical subjects). They were given the Children's Form of the Taylor Manifest Anxiety Scale, the Dutton's Scale of Attitudes Toward Arithmetic and each student was asked to list his or her five major subjects in order of preference.

The results of the study indicated that the anxiety level of the "achievers" was significantly higher than the "underachievers" and that the "achievers" had significantly more positive attitudes toward mathematics than the "underachievers". When ranking their major subjects in order of preference the "achievers" ranked mathematics significantly higher than the "underachievers". The following statement was included in Degnan's summary (11, p. 60):

The present findings tend to support the current view of several researchers in this area who contend that negative attitudes are associated with poor mathematics achievement among students whose achievement in other school subjects is average or above average. The present findings further support the view that anxiety plays a positive role in learning in that "achievers" were more anxious than "underachievers".

Degnan's study says nothing about cause and effect; is better achievement the result of favorable attitudes or vice versa? One would expect that each would contribute to the

other. Stephens (28) also conducted a study comparing attitudes and achievement of 348 seventh and eighth graders. Her findings were consistent with Degnan's in that it was determined that those who had accelerated in mathematics had significantly better attitudes toward mathematics than those who had not accelerated.

A similar hypothesis was tested by Aiken and Dreger (3) in their study to determine if attitude scores would make a significant contribution to the prediction of final grades in a mathematics course. Data on 60 males and 67 females taking general mathematics during their first year in college indicated that attitudes did contribute significantly to prediction of achievement for the female students but not for the male students. Data on 52 male and 63 female students in three other freshman level mathematics courses indicated that mathematics attitude scores predicted gains in scores on a mathematics achievement test.

Summary

The literature reviewed reveals that high school geometry has been the subject of debate among educators since the turn of the century. Research by Quast (22) traced the history of geometry in the secondary schools from 1890 to 1966. He reported that in spite of numerous recommendations for change, the formal, axiomatic development of geometry

has remained in most high schools for three-quarters of a century. Although modifications in geometry instruction have been minor, a number of new mathematics programs did appear during the reform years of 1955 to 1965. The effectiveness of the text materials of these new programs was investigated in the National Longitudinal Study of Mathematics Abilities (7). This five-year study revealed that the format or style in which a text is written does not contribute significantly to geometry achievement.

The remainder of the literature reviewed in this chapter was grouped into three categories: (1) Related Mathematical Structure Research, (2) Related Instructional Approach Research and (3) Related Mathematical Attitude Research.

Myers (17) studied some of the arguments made by proponents of the axiomatic-structure approach to geometry. Ability to think critically, the value of the knowledge of axiomatics, content mastery, and motivation are some of the issues he analyzed. Myers concluded that there is insufficient research evidence to support claims made that the axiomatic development of geometry makes a positive contribution to these issues. Yasui (35) compared achievement and attitudes of high school students who had studied for three years in a "modern" mathematics program with those who had not. He concluded that the modern program, which included more emphasis on structure, developed a greater

understanding of mathematics but had no effect on attitudes.

After reviewing what researchers have found and what others have written, it appears to this investigator that preferences for or against the axiomatic development of geometry are based primarily on tradition and personal feelings about why mathematics is studied rather than on experimental evidence. In recent years an increasing number of articles in educational journals are suggesting that geometry should not be used as the main vehicle for teaching the axiomatic development of mathematics.

In the "Related Instructional Approach" section of this chapter studies were reviewed that had made comparisons of teaching methods related to formal and informal instructional approaches. Kellogg (15) compared geometry instruction stressing deductive experiences, inductive experiences, and practical applications. No significant differences were noted on measures of attitudes and differences in achievement measures were noted, in favor of the deductive class, only on tests containing questions with deductive reasoning emphasis. Nichols (18) found that the inductive and deductive approaches of his study were equally effective in teaching plane geometry to high school freshmen. VanAkin (30) conducted a study to determine the relative effects of formal two-column proofs and less formal paragraph proofs on student knowledge of geometric facts and logical reason-

ing abilities. Neither method of proof provided a significant advantage in this experiment. Formal and informal presentations of selected geometry topics to seventh graders were compared by Bassler, Curry, Hall, and Mealy (6). No significant differences were obtained for immediate effects or for retention after six weeks. Zoll (36) investigated the relative merits of varying amounts of applications in geometry homework assignments. Data collected revealed no significant differences in knowledge of geometric facts or in the ability to make practical applications. There has been no conclusive evidence in these studies to suggest an advantage for the formal instructional approach over the informal, or vice versa.

The remaining studies reviewed relate to attitudes in mathematics education. Begle (7) reported that the five-year NLSMA study revealed that, starting at the junior high level, attitudes toward mathematics begin a steady decline until the end of high school. Anttonen (5) found that attitudes fluctuate considerably at the elementary level and begin to stabilize at the secondary level. Results of these two studies indicate that attitudes are especially important at the tenth grade level when students' feelings about mathematics are beginning to stabilize. Phelps (20) and Woodall (34) conducted studies to compare student attitudes in SMSG and traditional mathematics programs and found that, except

for the fifth grade level in Phelps' study, SMSG materials did not produce significant positive attitudes at grade levels four through eight. Poffenberger and Norton (21) involved college freshmen in a study to determine factors which contribute to favorable or unfavorable reactions toward mathematics. They found that attitudes and expectations of parents contributed significantly to their childrens' attitudes toward mathematics. Studies by Degnan (11), Stephens (28), and Aiken and Dreger (3) indicate a positive relationship between attitudes and mathematical achievement.

The research reviewed in this chapter presents no conclusive evidence favoring either formal or informal instructional approaches as they relate to geometry achievement or attitudes of students. Some writers have expressed the belief that emphasis on mathematical structure has a negative effect on attitudes and does not contribute to geometry achievement; these claims have not yet been verified. There is agreement that attitudes are important to mathematics education but research has not provided answers as to how favorable attitudes can be fostered in the mathematics classroom.

CHAPTER III. METHODS AND PROCEDURES

This study was designed to compare the effectiveness of the traditional axiomatic development of high school geometry with a less formal approach which placed less emphasis on mathematical structure and greater emphasis on practical applications. Achievement as it relates to the knowledge of geometric facts and relations, and attitude toward mathematics were the variables selected to assess the effectiveness of these two approaches.

This chapter describes the methods and procedures used to gather and analyze the data. This chapter has been divided into four sections: (1) Description of Courses and Materials, (2) Description of Testing Instruments, (3) Experimental Procedure and Collection of Data, (4) Treatment of Data.

Description of Courses and Materials

The geometry course studied by the control group is similar to that which has been taught in most high schools for several decades. The 1969 edition of Modern School Mathematics Geometry by Jurgensen, Donnelly, and Dolciani (14), a text designed for the axiomatic development of geometry, was used. Most of the traditional geometric topics were covered and are listed in Appendix A which gives that portion of the table of contents included in the course.

In the spirit of the text, the material was presented to the control classes with careful consideration given to the importance of definitions, postulates, theorems and logic so that the geometry developed would be classified as a mathematical system. Following the study of inductive and deductive reasoning and symbolic logic, early in the year, formal two-column proofs were introduced. Formal two-column proof here refers to the demonstration of the "truth" of a theorem by using a sequence of numbered statements in one column, with a reason to support each statement in an adjoining column. The reasons are restricted to definitions, postulates and previously proven theorems.

The lecture-demonstration method of presenting material was used in both experimental and control classes aided by the overhead projector and chalkboard. Students also used the chalkboard to demonstrate selected exercises to the rest of the class. A portion of the class period was used almost daily for supervised study. During this time students worked in small groups or individually on assigned homework. Each student was asked to keep homework assignments in a notebook to be handed in periodically.

Text materials for the experimental group were prepared by the instructors. In order to minimize any effect that the format or organization of the materials might have on the study, all materials except the career application units

were bound to form a textbook for each student. The table of contents can be seen in Appendix B and reveals that the text is organized into eleven units. Placed at the beginning of each unit is a list of definitions of special geometric terms needed for that unit and a list of the geometric properties to be studied. These geometric properties are designated as "facts" and are the same properties referred to as postulates and theorems in the control text. Following this are the practice exercises designed to lead the students, often through a discovery approach, to an understanding of the topics in the unit. With a few exceptions, solved examples as normally found in a commercially prepared text, are not provided. However, quite often a sequence of student exercises is provided to serve as an example for the student to follow when working succeeding problems.

The geometry studied in the experimental group differed in two significant ways. First, although the major topics included were identical to those covered in the control classes, the topics were introduced at a more concrete, operational level. The intent was to study a geometry of the real world rather than a pure mathematical geometry. A major objective was to develop the ability to recall, recognize, and use all the important facts, results, and formulas of the traditional geometry course without the restriction

of an abstract mathematical form. Intuitive, inductive and deductive reasoning were all used but no attempt was made to organize the entire year's work into one sequential deductive system. The question of why a certain geometric fact is true was often asked the student or discussed in class but no two-column formal proofs were made. Congruence of triangles is an example of a topic where informal proofs were used. Students were asked to specify the facts necessary to prove two triangles congruent in various triangular configurations. This came close to the traditional use of the Side-Angle-Side and Side-Side-Side postulates but with less rigor as far as the form of the student response was concerned.

The experimental geometry differed also in the emphasis placed on applications. During the second semester, career units were introduced to provide opportunities for students to apply what they had learned in realistic career situations. Twenty-two career units were made available to students from which they made selections according to their interests. The career units were obtained from the Minnesota State Department of Education and were written by Minnesota educators after on-the-job visits to determine the geometric properties actually being used in various occupations. These units were provided individual students in worksheet form on an independent study arrangement.

The instructors participating in this experiment are both experienced mathematics teachers; both hold the Masters Degree with a major in mathematics. Phil Johnson has taught high school geometry for fifteen years, including eight years at Ames Senior High School. Walter Wood (the investigator) has taught secondary mathematics for sixteen years, all at Ames Senior High School, with this being the second year for geometry instruction.

Description of Testing Instruments

Forms 111B and 117 of "The Ideas and Preferences Inventory" were used in this study to measure attitudes toward mathematics (31, pp. 13-22, 65-75). These instruments were developed for use in the National Longitudinal Study of Mathematical Abilities (NLSMA) which included over 112,000 students from 1,500 schools in 40 states (33, p. v). This five-year study (1962-65) was directed by the staff of the School Mathematics Study Group (SMSG). Permission to use these instruments was granted by Dr. Edward G. Begle, Director of SMSG. The two instruments consist of a number of subscales designed to measure various components of attitude toward mathematics and other personality variables related to motivation (33, p. 151). The following subscales were selected for use in this study (32, pp. 181-191):

1. MATH vs. NON-MATH (8 items) This scale is designed to measure how well a student likes mathematics and con-

siders it important in relation to other school subjects.

2. MATH FUN vs. DULL (4 items) This scale is designed to measure the pleasure or boredom a student experiences with regard to mathematics both in the absolute sense and comparatively with other subjects.

3. PRO-MATH COMPOSITE (11 items) This scale is designed to measure general attitude toward mathematics.

4. MATH EASY vs. HARD (9 items) This scale is designed to measure the ease or difficulty which a student associates with mathematics performance.

5. FACILITATING ANXIETY 2 (9 items) This scale is designed to measure the degree to which mathematics achievement performance is facilitated by stressful conditions (e.g., examinations).

6. DEBILITATING ANXIETY 2 (10 items) This scale is designed to measure the degree to which mathematics achievement performance is harmed by stressful conditions (e.g., examinations).

7. ACTUAL MATH SELF-CONCEPT (8 items) This scale is designed to measure how a child sees himself in relation to mathematics.

These seven subscales provide a Likert-type attitude scale of fifty-three items. The first four scales listed above measure attitudes toward mathematics, the fifth and sixth scales include items relevant to mathematics test anxiety, and the seventh is designed to assess the student's estimation of himself as a student. Extensive validation studies were carried out by the authors of these scales before they were used in the NLSMA study (33, pp. 151-5). The scales were administered to the same students at the ninth, tenth, and eleventh grade levels in the NLSMA study.

Statistics are provided for each scale from stratified random samples at grades nine and eleven. ALPHA coefficients, estimating the internal consistency reliability for each scale, ranged from 0.67 to 0.85 for grade nine and from 0.73 to 0.87 for grade eleven (32, pp. 241-51, 289-300). A copy of the instrument, as used in this study, can be seen in Appendix C.

The Geometry Attitude Survey (GAS-50) was developed by Felix G. Labaki (16). As far as this investigator could determine this is the only scale available which measures attitudes toward geometry specifically instead of toward mathematics in general. Permission was granted by Dr. Labaki for the GAS-50 to be used in this study. This instrument includes fifty Likert-style items and was developed to include the following five subscales (16, p. 71): Interest-Pleasure (13 items), Difficulty (8 items), Relevance (20 items), Comparison With Other Mathematics (3 items), and Teacher Influence (6 items). The Kuder-Richardson Reliability Coefficients were higher than 0.70 for the total scale and for all sub-scales except the Comparison With Other Mathematics sub-scale. This, along with other validation procedures, led Labaki to conclude (16, p. 80):

....it is reasonable to consider the scale as unidimensional, its basic referent being attitude toward geometry. At the same time, evidence is quite strong that at least four of the five identified sub-scales do measure attitudes toward their respective referents and

that consequently these separate factors can be considered as being encompassed globally by the overall referent of the total scale while still retaining their own identity.

To measure overall achievement of geometric content the Educational Testing Service (ETS) Cooperative Mathematics Tests: Geometry, Form A (1962) was selected. According to the ETS Handbook (10, pp. 55, 62):

An advisory committee of ten leaders in mathematics education was appointed ... to work with the ETS staff in developing specifications for the new series. ... 46 mathematics teachers, junior high school through college, were engaged to write the items. ... [after pre-tests and revisions] results indicated that these tests also were now appropriate for the intended populations.

Content validity is best insured by entrusting test construction to persons well-qualified to judge the relationship of test content to teaching objectives.

The ETS Handbook (10, p. 63) also reports a reliability coefficient of 0.87 for Form A of the geometry test computed using the Kuder-Richardson Formula #20 and a sample of 300. This is an eighty-item multiple choice test designed to measure "the important outcomes of a typical high school geometry course" including applications of geometry in three-dimensional space (10, p. 7). This test also includes six items on "logic and nature of proof".

Tests constructed by this investigator were also used as a measure of achievement. Whenever possible, common test items were used on unit examinations; two hundred questions were given to both experimental and control groups in this

way. In addition, forty-nine questions were given to all classes on the Semester I final examination.

Experimental Procedure and Collection of Data

This experiment was conducted during the 1974-75 school year with all tenth and eleventh grade geometry students at Ames Senior High School, Ames, Iowa participating. Approximately ninety-five tenth grade students were not included because they had accelerated their mathematics program in the eighth grade and studied geometry in grade nine. Standard procedure is for all students to pre-register during the second semester for the subjects they plan to take the following school year. Then immediately before school starts in the fall a "self-scheduling" day is designated for each student to choose a daily time schedule for his (or her) classes. In order to obtain a random assignment of geometry students, this "self-scheduling" procedure had to be altered somewhat for the students participating in this study.

During the summer months before the 1974-75 school year all students who had pre-registered for geometry were randomly assigned, with the aid of a table of random numbers, to experimental and control groups for their study of geometry. Specific class periods of the school day were then selected for the experimental and control groups. In an attempt to prevent the time of day from influencing the variables of

the study, periods were spaced throughout the day for both groups. To accommodate nine geometry classes, periods 1, 3, 4, 5, and 8 were assigned to the control group and periods 2, 4, 6, and 7 to the experimental group. All those who had pre-registered for geometry were notified by mail which periods were available for scheduling their geometry. There were some schedule conflicts on registration day that could not be resolved without changing from one group to the other but these were few in number and seemed to follow no pattern that could introduce a bias. There were also a few students who enrolled in geometry that had not pre-registered and vice versa so that the experiment started with 104 students in the experimental group and 119 students in the control group with nineteen in each group that had not originally been randomly assigned. The four experimental and five control classes were assigned to two instructors; Phil Johnson teaching periods 1(C), 3(C), 4(E), and 6(E) and this investigator teaching periods 2(E), 4(C), 5(C), 7(E), and 8(C) where the "E" and "C" indicate experimental and control classes.

I. Q. scores and mathematics grade point average for grades seven, eight and nine were obtained from school records for possible covariates in the statistical analysis. Pre-testing began on the second day of school when Part I of the ETS, Cooperative Mathematics Tests: Geometry, Form A

was given to all geometry students to measure content knowledge at the beginning of the experiment. On the tenth day of the school year the "Ideas and Preferences Inventory" (Appendix C) was administered to all geometry students to provide a measure of student attitudes toward mathematics before their study of geometry. It was this investigator's opinion that by waiting ten days, students would have time to realize they were participating in a geometry experiment and any effect this might have on their feelings toward mathematics would be reflected in each administration of the instrument and therefore would not contribute to any measure of change in attitude.

As the experiment progressed the two participating instructors cooperated in constructing unit tests and giving the same assignments so that all experimental classes were given the same unit test on the same day and likewise for the control classes. Since instructional emphases were different and topics were not studied in the same order, different unit tests were used for the two treatment groups. However, as many common test items as possible were placed on every test given throughout the year. Two hundred forty-nine common questions were used on these teacher-made examinations.

One week before the end of the first semester of the school year "The Ideas and Preferences Inventory" was admin-

istered for the second time to all geometry classes to measure any change in attitudes toward mathematics. There were two major reasons for measuring attitudes at this time. First, the control group had by this time been constructing two-column formal proofs for approximately eleven weeks and the chapters to be covered during the second semester would include less emphasis on proof. Secondly, the experimental group was to begin the second semester with a nine-week emphasis on practical applications of geometry in career-related units.

At the end of Semester I the control classes had completed chapters one through seven and the first three sections of chapter eight in Modern School Mathematics, Geometry by Jurgensen, Donnelly, and Dolciani (see Appendix A). The experimental classes had completed units one through ten in their text (see Appendix B).

During the thirteenth week of the second semester "The Ideas and Preferences Inventory" was administered for the third time. This marked the time in the experiment when the scheduled experiences for the experimental and control groups would begin to be the same. The text materials prepared for the experimental group had been completed and each student in the experimental group was issued a copy of the textbook being used by the control group. During the next three weeks all classes received the same treatment.

The chapters "Coordinate Geometry-Methods" and "Coordinate Geometry-Proofs" were studied with emphasis given to constructing coordinate proofs. There were two major reasons for finishing the year with this unit on coordinate geometry. First, it was the opinion of the two instructors that students in both groups needed an introduction to this type of proof for subsequent mathematics courses. Secondly, it was the desire of this investigator to determine if those who had studied geometry without emphasis on proofs (experimental group) and those that had had considerable work with formal proofs (control group) would perform significantly different on this unit with coordinate proof emphasis.

The one week remaining in the school year was spent reviewing with no review exercises assigned that related to proofs. After the second day of review, the "Geometry Attitude Survey" was administered to all students. This instrument (see Appendix D) measures attitudes toward geometry specifically instead of toward mathematics in general as the instrument used earlier had done.

During one of the last two days of the school year each geometry class was administered the ETS, Cooperative Mathematics Tests: Geometry, Form A during a ninety minute final examination period.

Treatment of Data

Data collected for this experiment were coded and transferred to IBM cards so that all calculations and statistical analyses could be done by computer at the Iowa State University Computation Center. The system of computer programs, Statistical Package for the Social Sciences (SPSS), was used to provide the necessary statistical procedures (19).

The purpose of this experiment was to assess the effectiveness of two approaches to the teaching of tenth grade geometry. Success was measured by two criterion variables: achievement and attitudes. Experimental and control groups were formed by random assignment of students. Before analyzing data collected for treatment effects tests were made to determine whether the two groups were biased on selected variables thought to be related to geometry success. T-tests were used to compare means for the two groups on the following variables: IQ, grade-point average for seventh and eighth grade mathematics, first year algebra grade, score on a geometry achievement pre-test, and scores on eight subscales of an attitude toward mathematics pre-test. Pearson product-moment correlation coefficients were also computed for all pairs of variables including both pre- and post-tests. Stepwise regression of post-test achievement scores on independent variables IQ, seventh and eighth mathematics GPA, and first year algebra grade was employed to determine

the contribution made by each of these variables toward explaining the variance in achievement scores.

After examining these statistical measures it was determined that the difference in mean IQ values for the experimental and control groups was significant and should be statistically controlled in achievement data analysis. The following table reveals the t-test results for the IQ variable.

Table 1. T-Tests for IQ Scores

Group	Semester I				Semester II			
	N	Mean	t-value	Prob.	N	Mean	t-value	Prob.
Experimental	100	111.8			95	112.1		
Control	108	114.4	1.92	.056	97	115.2	2.26	.025*

*Significant at $P < 0.05$

The multiple regression method of analysis of covariance was used to analyze all achievement data for group comparisons. For this technique the categorical variable representing experimental and control groups is inserted into the regression equation in the form of a dummy variable. Through this dummy variable each group represented by the categorical variable is treated as a separate variable. Regression coef-

ficients are produced to form a prediction equation for each group and necessary statistics can be extracted from the computer output to compare means of the groups, after adjustment for the covariate, and to test for covariate-by-factor interaction. A detailed explanation of this procedure is given by J. Kim and F. Kohout in *Statistical Package for the Social Sciences* (19, pp. 373-383). The regression model for this procedure can be written as

$$Y' = A + B_1D + B_2X + B_3DX$$

and when $D = 1$ is assigned to the experimental group and $D = 0$ assigned to the control group, the following prediction equations are produced:

$$Y'_{\text{control}} = A + B_2X$$

$$Y'_{\text{experimental}} = (A + B_1) + (B_2 + B_3)X$$

Each letter in this model is defined as follows:

Y' = predicted value of the criterion variable

D = dummy variable representing experimental and control groups; $D = 1$ for experimental group and $D = 0$ for control group

X = covariate (IQ)

DX = covariate-by-factor interaction variable

A = constant term, the Y intercept of the control group regression line

B_1 = distance between Y intercepts of experimental and control group regression lines

B_2 = slope of control group regression line

B_3 = indicator of interaction effect

The first question to be answered from the examination of this multiple regression model is whether there exists a significant interaction between the achievement criterion variable and the covariate, IQ. This is equivalent to testing for homogeneity of slopes of regression lines for the experimental and control groups. If these slopes do not differ significantly then tests for main effects can be made.

Hypothesis testing with this analysis of covariance scheme is summarized in Table 2 (see page 58). Computer output from two regression runs provide the necessary statistics to construct the table. This procedure was used for comparisons on the following criterion measures:

1. Semester I total score for test items given to both both groups on teacher-made unit tests and first semester final examination.
2. Yearly total score for test items given to both groups on all teacher-made tests except a coordinate geometry unit.
3. Coordinate geometry unit test score.
4. Coordinate geometry proof test score.
5. ETS Cooperative Mathematics Geometry Test score

with logic and proof questions removed.

6. ETS Cooperative Mathematics Geometry Test total score.

Table 2. Analysis of Covariance

Sources of Variation	Sums of Squares	F-Ratio
(1) SS due to Z^a and X^b , saturated model	$SS_Y[R_{Z,X,ZX}^2]^c$	$\frac{(1)/3}{(4)/N-4}$
(2) SS due to Z and X, additive model	$SS_Y[R_{Z,X}^2]$	$\frac{(2)/2}{(4)/N-4}$
(a) SS due to Z, adjusted for X	$SS_Y[R_{Z,X}^2 - R_X^2]$	$\frac{(2a)}{(4)/N-4}$
(b) SS due to X, adjusted for Z	$SS_Y[R_{Z,X}^2 - R_Z^2]$	$\frac{(2b)}{(4)/N-4}$
(3) SS due to interaction, lack of homogeneity of slopes	$SS_Y[R_{Z,X,DX}^2 - R_{Z,X}^2]$	$\frac{(3)}{(4)/N-4}$
(4) SS residual	$SS_Y[1 - R_{Z,X,ZX}^2]$	

^a Z = method of instruction variable (experimental and control groups). It is represented by the dummy variable, D, in the regression model.

^b X = covariate.

^c $R_{Z,X,ZX}^2$ = the multiple correlation of the saturated model. The product $SS_Y[R_{Z,X,ZX}^2]$ is the sum of squares explained by both main and interaction effects.

The NLSMA attitude toward mathematics scales were administered early in the school year to provide a base for measuring any change in attitudes as the experiment progressed. T-tests comparing experimental and control groups revealed no significant differences in group means on any of the attitude pre-test scores. The Pearson product-moment correlation coefficients revealed no strong relationships between attitude scales and other pre-test measures. It was concluded that no covariates would be used on subsequent analyses of attitude data.

All attitude scales given at the beginning of the school year were given a second time at the end of the first semester and a third time near the end of the experiment. Change scores for each student on each scale were recorded and t-test comparisons of group means were used to detect any significant changes in attitudes toward mathematics. T-tests were also used to compare groups on the Geometry Attitude Survey (GAS-50) administered at the end of the experiment.

In addition to comparisons of group means based on achievement and attitude scores, as described above, tests were also made to locate possible interaction effects between instructional methods and independent variables IQ, mathematics grade-point average, and sex of student. In order to detect interaction between categorical variables representing instructional method and sex, the standard

analysis of variance or covariance procedure was used as described by Kim and Kohout in *Statistical Package for the Social Sciences* (19, pp. 398-422). In order to assess the relationship of the instructional methods to metric variables IQ and grade-point average, the method described by Ferguson (13, pp. 187-8) was used to determine the significance of the difference between two correlation coefficients. First, Pearson product-moment correlation coefficients (r) were computed between a dependent and an independent variable for both experimental and control groups. The two r 's were then converted into Z_r -values by using Fisher's Z_r transformation (13, p. 412). Next, the standard error of the difference between two values of Z_r was computed by the following formula:

$$S_{Z_1-Z_2} = \sqrt{1/(N_1-3) + 1/(N_2-3)} .$$

When the difference between

the two values of Z_r is divided by the standard error of the

difference, the following ratio is formed: $z = \frac{Z_1 - Z_2}{S_{Z_1-Z_2}} .$

This is a unit-normal-curve variate and values of 1.96 and 2.58 are required for significance at the .05 and .01 levels, respectively.

CHAPTER IV. FINDINGS

The major concern of this chapter is to report the statistical treatment of data and to examine the data for evidence which either supports or contradicts the null hypotheses stated in Chapter I. Each hypothesis will be re-stated, followed by an analysis of data, and then a decision as to whether there is sufficient evidence to reject the null hypothesis.

Hypothesis 1: There is no significant difference in group means of students who have studied geometry under an informal instructional approach with career-oriented units (experimental) and those who have studied under an axiomatic, formal-proof approach (control) when tested on achievement in traditional geometric properties by teacher-made unit examinations.

Table 3 depicts the mean scores on teacher-made unit tests for both the experimental and control groups. The scores reflect the number of common test items worked correctly. Unit tests were not identical for both groups but, when possible, common test items were included on experimental and control unit tests. Table 3 also shows group means after they have been adjusted for the covariate, IQ.

Table 4 summarizes the multiple regression analysis of covariance for first semester achievement. This model was discussed in the previous chapter.

Table 3. Means for common test items on teacher-made tests

Variate	Group	N	Mean	Std. Dev.	Adjusted ^a Mean
Sem. I common test items	Experimental	100	100.91	17.45	102.14
	Control	108	102.93	19.90	101.79
Full-year common test items	Experimental	95	149.65	24.94	151.52
	Control	97	156.73	25.60	154.91

^aAdjusted Mean = Mean adjusted for group differences on the covariate, IQ.

Table 4. Analysis of Covariance for Semester I teacher-made tests

Source of Variation	Sums of Squares	df	F-ratio ^a
(1) Saturated Model	SS _Y (.22809)	3	20.093**
(2) Additive Model	SS _Y (.22796)	2	30.123**
Method, adjusted for IQ	SS _Y (.00008)	1	.021
IQ, adjusted for Method	SS _Y (.22503)	1	59.471**
(3) Interaction, lack of homogeneity of slopes	SS _Y (.00016)	1	.042
(4) Residual	SS _Y (.77191)	204	

^a F_{.05}(3,204) = 2.65 F_{.01}(3,204) = 3.88
 F_{.05}(2,204) = 3.04 F_{.01}(2,204) = 4.71
 F_{.05}(1,204) = 3.89 F_{.01}(1,204) = 6.76

**Significant at P<0.01.

The F-ratio of .042 for interaction effect in Table 4 indicates that the assumption of homogeneity of slopes for experimental and control group regression lines is valid and therefore this model provides a meaningful test for main effects. The F-ratio of .021 for method effect is well below that required at the .05 level (3.89). Hence, there is insufficient evidence to reject Hypothesis 1, at the .05 level of significance, based on semester one unit tests.

Table 5. Analysis of Covariance for total teacher-made unit tests (coordinate geometry excluded)

Source of Variation	Sums of Squares	df	F-ratio ^a
(1) Saturated Model	SS _Y (.21687)	3	17.354**
(2) Additive Model	SS _Y (.21626)	2	25.958**
Method, adjusted for IQ	SS _Y (.00434)	1	1.042
IQ, adjusted for Method	SS _Y (.19683)	1	47.251**
(3) Interaction, lack of homogeneity of slopes	SS _Y (.00061)	1	.146
(4) Residual	SS _Y (.78313)	188	

^a F _{.05} (3,188) = 2.65	F _{.01} (3,188) = 3.89
F _{.05} (2,188) = 3.04	F _{.01} (2,188) = 4.72
F _{.05} (1,188) = 3.89	F _{.01} (1,188) = 6.77

**Significant at P<0.01.

Table 5 reveals the multiple regression analysis of covariance for common test items on all teacher-made unit tests administered during the comparison of the two instructional methods. The last unit of the school year, coordinate geometry with proofs, is not included in this table since the experimental and control groups received identical treatment during the study of this unit. Data from the coordinate geometry unit will be analyzed separately. The F-ratio of .146 indicates no significant interaction effect and the F-ratio of 1.042 is below the .05 significance level (3.89) for method effect.

Tables 4 and 5 provide insufficient evidence to reject Hypothesis 1.

Hypothesis 2: There is no significant difference between mean scores of the experimental and control groups when given a standardized, full-year geometry achievement test.

Table 6 depicts the mean scores for experimental and control groups on the ETS Cooperative Mathematics Geometry Test, Form A. In this table also are mean subtest scores on six items categorized in the ETS handbook (10, p. 30) as "logic and nature of proof" items and mean subtest scores on the remaining seventy-four items. These subtest scores are included because a major difference in the treatment received by the two groups involved the use of formal proofs of geometry theorems. The experimental group made no formal

two-column proofs during their study of geometry, whereas formal proofs were an essential part of geometry for the control group.

Table 6. Means for ETS Cooperative Mathematics geometry achievement test

Variate	Group	N	Mean	Std. Dev.	Adjusted ^a Mean
Logic and proof items	Experimental	95	2.69	1.20	2.71
	Control	97	3.24	1.33	3.22
Total test, logic and proof items excluded	Experimental	95	39.29	8.49	40.00
	Control	97	41.94	8.95	41.25
Total test	Experimental	95	41.99	8.78	42.71
	Control	97	45.18	9.42	44.47

^aAdjusted Mean = Mean adjusted for group differences on the covariate IQ.

After adjustment for IQ differences the experimental group mean for the six "logic and nature of proof" items was 2.71 and for the control group it was 3.22. Table 7 contains a summary of the multiple regression analysis of covariance for this variable. The assumption of homogeneity of slopes, with an F-ratio of .728, is tenable. The F-ratio for method or group effect is 7.510 which is significant beyond the .01 level of significance (6.77). The difference in group means for the six "logic and nature of proof" items is statistically significant in favor of the control group.

Table 7. Analysis of Covariance for "logic and nature of proof" items on the ETS Cooperative Mathematics geometry achievement test

Source of Variation	Sums of Squares	df	F-ratio ^a
(1) Saturated Model	SS _Y (.06203)	3	4.144**
(2) Additive Model	SS _Y (.05840)	2	5.853**
Method, adjusted for IQ	SS _Y (.03747)	1	7.510**
IQ, adjusted for Method	SS _Y (.01242)	1	2.489
(3) Interaction, lack of homogeneity of slopes	SS _Y (.00363)	1	.728
(4) Residual	SS _Y (.93797)	188	

$$\begin{array}{ll}
 {}^a F_{.05}(3,188) = 2.65 & F_{.01}(3,188) = 3.89 \\
 F_{.05}(2,188) = 3.04 & F_{.01}(2,188) = 4.72 \\
 F_{.05}(1,188) = 3.89 & F_{.01}(1,188) = 6.77
 \end{array}$$

**Significant at $P < 0.01$.

Table 8 shows the analysis of covariance for the remaining seventy-four items on the ETS geometry achievement test. After adjustment for IQ, the means for this variable were 40.00 for the experimental group and 41.25 for the control group. With an F-ratio of .183, the test for homogeneity of slopes is satisfied and the F-ratio of 1.250 for method effects is not significant at the .05 level. This analysis then does not indicate a significant difference in performance on the ETS geometry achievement test when the

"logic and nature of proof" items are excluded.

Table 8. Analysis of Covariance for the ETS Cooperative Mathematics geometry achievement test, "logic and nature of proof" items excluded

Source of Variation	Sums of Squares	df	F-ratio ^a
(1) Saturated Model	SS _Y (.25877)	3	21.877**
(2) Additive Model	SS _Y (.25805)	2	32.725**
Method, adjusted for IQ	SS _Y (.00493)	1	1.250
IQ, adjusted for Method	SS _Y (.23538)	1	59.700**
(3) Interaction, lack of homogeneity of slopes	SS _Y (.00072)	1	.183
(4) Residual	SS _Y (.74123)	188	

^a F _{.05} (3,188) = 2.65	F _{.01} (3,188) = 3.89
F _{.05} (2,188) = 3.04	F _{.01} (2,188) = 4.72
F _{.05} (1,188) = 3.89	F _{.01} (1,188) = 6.77

**Significant at P<0.01.

Tables 7 and 8 provide information relative to the nature of possible group differences on the ETS geometry achievement test by showing a significant difference on items dealing with logic and nature of proof and a lack of support for a significant difference on the remaining items. However, Hypothesis 2 refers to a performance comparison on the entire test, so the tenability of Hypothesis 2 will be judged

on this basis. Table 9 summarizes the analysis of covariance for the entire ETS geometry test. The means after adjustment for IQ were 42.71 for the experimental group and 44.47 for the control group.

Table 9. Analysis of Covariance for the total ETS Cooperative Mathematics geometry achievement test.

Source of Variation	Sums of Squares	df	F-ratio ^a
(1) Saturated Model	$SS_Y(.26027)$	3	22.049**
(2) Additive Model	$SS_Y(.25911)$	2	32.926**
Method, adjusted for IQ	$SS_Y(.00885)$	1	2.249
IQ, adjusted for Method	$SS_Y(.22894)$	1	58.184**
(3) Interaction, lack of homogeneity of slopes	$SS_Y(.00116)$	1	.295
(4) Residual	$SS_Y(.73973)$	188	

$$\begin{array}{ll}
 {}^a F_{.05}(3,188) = 2.65 & F_{.01}(3,188) = 3.89 \\
 F_{.05}(2,188) = 3.04 & F_{.01}(2,188) = 4.72 \\
 F_{.05}(1,188) = 3.89 & F_{.01}(1,188) = 6.77
 \end{array}$$

**Significant at $P < 0.01$.

The F-ratio of .295 indicates the assumption of homogeneity of slopes is acceptable and the F-ratio of 2.249 for method is not significant at the .05 level (3.89). There is insufficient evidence to reject Hypothesis 2.

Hypothesis 3: There is no significant difference in performance of the experimental and control groups, as measured by a teacher-made achievement test, when a coordinate geometry unit is studied that includes coordinate geometry proofs.

Coordinate geometry was the last unit studied in this experiment. Both experimental and control groups received the same treatment on this unit. The tests over the unit included seven coordinate geometry proofs. Table 10 includes group means on these seven proofs and group means on total test scores.

Table 10. Means for the teacher-made coordinate geometry unit achievement tests

Variate	Group	N	Mean	Std. Dev.	Adjusted ^a Mean
Proofs subtest	Experimental	95	3.48	2.19	3.59
	Control	97	3.98	2.18	3.86
Total test	Experimental	95	20.97	6.09	21.20
	Control	97	22.42	5.33	22.19

^aAdjusted Mean = Mean adjusted for group differences on the covariate IQ.

Table 11 presents the analysis of covariance for the seven items on the proofs subtest. The adjusted experimental group mean on this subtest was 3.59 and for the control group it was 3.86.

Table 11. Analysis of Covariance for the proof subscale of the teacher-made coordinate geometry achievement tests

Source of Variation	Sums of Squares	df	F-ratio ^a
(1) Saturated Model	SS _Y (.11196)	3	7.901**
(2) Additive Model	SS _Y (.10978)	2	11.620**
Method, adjusted for IQ	SS _Y (.00378)	1	.800
IQ, adjusted for Method	SS _Y (.09695)	1	20.525**
(3) Interaction, lack of homogeneity of slopes	SS _Y (.00218)	1	.462
(4) Residual	SS _Y (.88804)	188	

$$\begin{array}{ll}
 {}^a F_{.05}(3,188) = 2.65 & F_{.01}(3,188) = 3.89 \\
 F_{.05}(2,188) = 3.04 & F_{.01}(2,188) = 4.72 \\
 F_{.05}(1,188) = 3.89 & F_{.01}(1,188) = 6.77
 \end{array}$$

**Significant at $P < 0.01$.

The homogeneity of slopes assumption is satisfied with an F-ratio of .462 and the method F-ratio of .800 is not significant at the .05 level (3.89). Evidence provided by this analysis does not suggest a significant difference in group means for the coordinate proof items.

The analysis of total test scores for the coordinate geometry unit is shown in Table 12. The adjusted means are 21.20 for the experimental group and 22.19 for the control group for this variable.

Table 12. Analysis of Covariance for the teacher-made coordinate geometry achievement tests total

Source of Variation	Sums of Squares	df	F-ratio ^a
(1) Saturated Model	SS _Y (.08398)	3	5.745**
(2) Additive Model	SS _Y (.07780)	2	7.984**
Method, adjusted for IQ	SS _Y (.00722)	1	1.482
IQ, adjusted for Method	SS _Y (.06172)	1	12.667**
(3) Interaction, lack of homogeneity of slopes	SS _Y (.00618)	1	1.268
(4) Residual	SS _Y (.91602)	188	

$$\begin{array}{ll}
 {}^a F_{.05}(3,188) = 2.65 & F_{.01}(3,188) = 3.89 \\
 F_{.05}(2,188) = 3.04 & F_{.01}(2,188) = 4.72 \\
 F_{.05}(1,188) = 3.89 & F_{.01}(1,188) = 6.77
 \end{array}$$

**Significant at $P < 0.01$.

The interaction F-ratio (1.268) does not indicate a significant interaction and the method F-ratio (1.482) does not indicate a significant difference in group means due to instructional method. There is insufficient evidence to reject Hypothesis 3.

Hypothesis 4: There is no significant difference between mean scores of the experimental and control groups when given an attitude-toward-mathematics scale.

Scores in Table 13 are from The Ideas and Preferences Inventory (31), an attitude-toward-mathematics scale which

includes seven subscales described in Chapter III. This instrument was administered three times during the study and scores in Table 13 are mean changes measured at the end of Semester I and again near the end of the school year. Also included in Table 13 are the results of t-tests for each subscale testing the significance of the difference in experimental and control group means.

Table 13. Group means and T-Tests for the Ideas and Preferences Inventory, an attitude-toward-mathematics scale

Variate	N		Means		t-value	2-tail Prob.
	Exp.	Control	Exp.	Control		
Math vs. Non-Math						
(a) Sem. I change	102	117	0.446	-0.483	1.95	0.053
(b) Sem. II change	98	106	-0.500	-0.203	-0.69	0.493
(c) Total change	97	106	0.005	-0.439	0.86	0.393
Fun vs. Dull						
(a) Sem. I change	102	117	0.485	-0.432	2.04	0.043*
(b) Sem. II change	98	106	-0.107	0.241	-0.89	0.373
(c) Total change	97	106	0.464	-0.123	1.30	0.195
Composite						
(a) Sem. I change	102	117	0.446	0.094	0.59	0.558
(b) Sem. II change	98	106	-0.276	0.014	-0.52	0.602
(c) Total change	97	106	0.284	0.368	-0.13	0.897
Easy vs. Hard						
(a) Sem. I change	102	117	1.466	0.684	1.16	0.247
(b) Sem. II change	98	106	-0.546	0.019	-1.01	0.311
(c) Total change	97	106	1.134	0.745	0.53	0.595

*Significant at $P < 0.05$.

Table 13. (Cont'd.)

Variate	N		Means		t-value	2-tail Prob.
	Exp.	Control	Exp.	Control		
Math Attitude Sub-Total^a						
(a) Sem. I change	102	117	2.843	-0.137	1.76	0.080
(b) Sem. II change	98	106	-1.429	0.071	-1.01	0.313
(c) Total change	97	106	1.887	0.552	0.70	0.484
Facilitating Anxiety						
(a) Sem. I change	102	117	0.299	-0.962	2.36	0.019*
(b) Sem. II change	98	106	-0.357	1.500	-3.41	0.001**
(c) Total change	97	106	-0.119	0.533	-1.18	0.239
Debilitating Anxiety						
(a) Sem. I change	102	117	0.265	0.124	0.21	0.837
(b) Sem. II change	98	106	-0.566	0.481	-1.62	0.106
(c) Total change	97	106	-0.180	0.835	-1.36	0.177
Self-Concept						
(a) Sem. I change	102	117	1.284	1.727	-0.49	0.627
(b) Sem. II change	98	106	-0.444	0.491	-1.16	0.248
(c) Total change	97	106	-1.263	-0.888	-0.40	0.689
Math Attitude Total^b						
(a) Sem. I change	102	117	4.691	0.753	1.94	0.054
(b) Sem. II change	98	106	-2.796	2.543	-2.10	0.037*
(c) Total change	97	106	0.325	1.032	-0.22	0.823

^aMath Attitude Sub-Total scores were obtained by summing scores from four subscales: Math vs. Non-Math, Fun vs. Dull, Composite, and Easy vs. Hard.

^bMath Attitude Total scores were obtained by summing scores from all seven subscales.

**Significant at $P < 0.01$.

Four t-tests recorded in Table 13 indicate significance beyond the .05 level. Group means for Semester I change scores on the Fun vs. Dull subscale differ significantly ($P = .043$) in favor of the experimental group. This subscale consists of eight items and "is designed to measure the pleasure or boredom a student experiences with regard to mathematics both in the absolute sense and comparatively with other subjects" (32, p. 182). The mean change for the experimental group was +0.485 and the mean change for the control group was -0.432.

The t-tests for difference in group means on the Facilitating Anxiety subscale indicate a significant change ($P = .019$) in favor of the experimental group for Semester I and a significant change ($P = .001$) in favor of the control group for Semester II. The mean Semester I change for the experimental group was +0.299 while it was -0.962 for the control group. The mean Semester II change was -0.357 for the experimental group and +1.500 for the control group. The total change for the year was not significant at the .05 level. The Facilitating Anxiety scale consists of nine items and is "designed to measure the degree to which mathematics achievement performance is facilitated by stressful conditions (e.g., examinations)" (32, p. 189).

The t-test for the Math Attitude Total scale also indicates a significant difference ($P = .037$) for second semester

group means. Scores for this scale were obtained by summing the change scores on all seven subscales and is therefore an indication of the accumulative effect of attitude changes on all subscales. The change in attitude on this total scale favors the control group for the second semester. Semester II mean change was -2.796 for the experimental group and $+2.543$ for the control group. The t-test on this scale also indicates that the yearly change for the entire attitude scale (including all seven subscales) did not differ at the .05 level of significance.

Two other t-tests indicate mean differences that are significant at the .05 level when probabilities are rounded to two decimal places. The mean change for the experimental group on the Math vs. Non-Math subscale was $+2.446$ while the change for the control group was -0.483 . The difference between these changes is significant at the .053 level. The Math vs. Non-Math subscale measures "how well a student likes mathematics and considers it important in relation to other school subjects" (32, p. 181). Also, the semester one mean change on the Math Attitude Total scale was $+4.691$ for the experimental group and $+0.753$ for the control group. The difference in these changes is significant at the .054 level.

When t-test results are examined for semester changes on individual subscales, significance at or beyond the .05 level is indicated on seven different occasions. But when

total changes for the year are examined, there is no indication of significance at the .05 level. Hence, there is insufficient evidence to reject Hypothesis 4.

Hypothesis 5: There is no significant difference between mean scores of the experimental and control groups when given an attitude-toward-geometry scale.

Scores in Table 14 are from the Geometry Attitude Survey (16) measuring attitudes toward geometry specifically rather than toward mathematics in general. In addition to total scores on this instrument, means for three subscales are also recorded: Interest Pleasure (13 items), Difficulty (8 items), and Relevance (20 items). The total scale includes fifty items. Table 14 reveals the results of t-tests for differences in group means on each of these scales.

Table 14. Group means and T-Tests for the Geometry Attitude Survey

Variate	N		Means		t-value	2-tail Prob.
	Exp.	Control	Exp.	Control		
Interest-Pleasure	97	106	36.91	37.08	-0.19	0.847
Difficulty	97	106	25.62	26.01	-0.55	0.584
Relevance	97	106	69.40	68.72	0.47	0.642
Total	97	106	159.06	159.37	-0.10	0.922

This scale was administered at the end of the experiment and provides a comparison of students' feelings toward the two geometry courses. The analysis of geometry attitude scores summarized in Table 14 reveals no significant differences in means for any of the subscales nor for the total scale. There is insufficient evidence to reject Hypothesis 5 at the .05 level of significance.

Hypothesis 6: There is no significant difference in the relationship of achievement and IQ scores for the experimental and control groups.

To test Hypothesis 6 the significance of the difference between two Pearson product-moment correlation coefficients was computed as described in Chapter III. Using the tabled values for Fisher's Z_r transformation, the coefficients of correlation for the experimental and control groups are converted to Z_r 's. The difference of these two Z -values are then divided by the standard error of the difference (i.e. $S_{Z_{r1}-Z_{r2}} = \sqrt{1/(N_1-3) + 1/(N_2-3)}$). The critical values for this ratio are 2.58 and 1.96 at the .05 and .01 significance levels respectfully (13, pp. 187-8). The results of this test are shown in Table 15 for total scores on teacher-made unit tests and the ETS Cooperative Mathematics geometry achievement test. The analysis shown in this table is an indication of whether the relationship between achievement scores and IQ scores is significantly different for the

experimental and control groups. This analysis is also an indication of whether there is an interaction between instructional method and IQ level of the student.

Table 15. Comparison of coefficients of correlation for achievement and IQ

Achievement Variable	Group	N	r	Fisher's Z	$S_{Z_1-Z_2}$	CR
Teacher-made unit tests	Experimental	95	.4850	.530	.147	.619
	Control	97	.4122	.439		
ETS achievement test	Experimental	95	.4746	.516	.147	.204
	Control	97	.4975	.546		

All correlation coefficients in Table 14 indicate a positive relationship between achievement and IQ. The observed value of the critical ratio (CR) for teacher-made tests and IQ is .619 and for the ETS Cooperative Mathematics geometry achievement test and IQ it is .204. Both are well below significance at the .05 level (CR = 1.96). Thus, there is insufficient evidence to reject Hypothesis 6.

Hypothesis 7: There is no significant difference in the relationship of geometry achievement scores and grades seven through nine mathematics grade-point averages for the experimental and control groups.

The results of the test for significance of difference between coefficients of correlation for geometry achievement and mathematics grade-point average is shown in Table 16.

Table 16. Comparison of coefficients of correlation for achievement and mathematics grade-point average

Achievement Variable	Group	N	r	Fisher's Z	$S_{Z_1-Z_2}$	CR
Teacher-made unit tests	Experimental	97	.5132	.517	.144	.903
	Control	103	.6049	.701		
ETS achievement test	Experimental	98	.3403	.354	.143	1.23
	Control	103	.4853	.530		

Correlation coefficients in Table 16 indicate a positive relationship between mathematics grade-point average and geometry achievement for both experimental and control groups. The test for significance of difference between correlation coefficients after Fisher's Z_r transformations yields a CR of .963 for teacher-made unit tests and 1.23 for the ETS Cooperative Mathematics geometry achievement test. These are both below the required CR (1.96) and consequently neither is significant at the .05 level. There is insufficient evidence to reject Hypothesis 7.

Hypothesis 8: There is no significant difference in the relationship of scores from attitude scales and IQ scores for the experimental and control groups.

Coefficients of correlation were computed for both experimental and control groups between total scores from the Ideas and Preferences Inventory, an attitude-toward-mathematics scale, and IQ scores and also between total scores from the Geometry Attitude Survey and IQ scores.

Table 17 shows the results of the significant difference tests using Fisher's Z_r transformation.

Table 17. Comparison of coefficients of correlation for attitude scales and IQ

Achievement Variable	Group	N	r	Fisher's Z	$S_{Z_1-Z_2}$	CR
Attitude toward mathematics	Experimental	95	.3222	.334	.147	2.34*
	Control	97	-.0103	.010		
Attitude toward geometry	Experimental	94	.1743	.176	.147	.381
	Control	97	.1194	.120		

*Significant at $P < 0.05$.

All coefficients of correlation in Table 17 are positive except for the control group correlation of attitude-toward-mathematics and IQ (-.0103). This is not a strong negative relationship but when compared with a positive correlation of .3222 for the experimental group, the CR value of 2.34 is beyond the required at the .05 level of significance (1.96). This indicates a significant difference between the experimental and control groups in the correlation of attitudes toward mathematics and IQ. Hypothesis 8 is rejected at the .05 level of significance. It should also be noted that from the analysis in Table 17 there is no indication of a significant difference between the two groups in the correlation of attitudes toward geometry and IQ scores.

Hypothesis 9: There is no significant difference in the relationship of scores from attitude scales and grades seven through nine mathematics grade-point averages for the experimental and control groups.

Table 18 depicts the results of tests for significant differences in correlation coefficients referred to in Hypothesis 9. Correlation coefficients were computed relating mathematics grade-point average to attitudes toward mathematics in general, and toward geometry specifically, using scores from the Ideas and Preferences Inventory and Geometry Attitude Survey, respectively.

Table 18. Comparison of coefficients of correlation for attitude scores and mathematics grade-point average

Achievement Variable	Group	N	r	Fisher's Z	$S_{Z_1-Z_2}$	CR
Attitude toward mathematics	Experimental	97	-.0690	.069	.144	.667
	Control	103	.0268	.027		
Attitude toward geometry	Experimental	97	.0698	.070	.144	1.056
	Control	103	.2184	.222		

The observed CR values of .667 and 1.056 shown in Table 18 are below the required CR (1.96) and thus are not significant at the .05 level. There is insufficient evidence to reject Hypothesis 9.

Hypothesis 10: When achievement is used as the criterion variable, there is no significant interaction between instructional method and sex of the student.

The standard two-way analysis of covariance, with IQ as the covariate, was used to test Hypothesis 10. Table 19 reveals the F-ratios corresponding to adjusted variance sums of squares for group by sex interaction when the experimental and control groups are compared on achievement scores.

Table 19. Analysis of Covariance, group by sex interaction, based on geometry achievement scores

Achievement Variable	Covariate	df	F-ratio	Significance Level
Teacher-made unit tests	IQ	(1,187)	1.145	0.286
ETS Cooperative Math. test	IQ	(1,187)	2.294	0.128

As Table 19 reveals, the significance levels of .286 for teacher-made tests and .128 for the ETS geometry achievement test do not support rejection of the null hypothesis. There is insufficient evidence to reject Hypothesis 10 at the .05 significance level.

Hypothesis 11: When attitude is used as the criterion variable, there is no significant interaction between instructional method and sex of the student.

Table 20 shows the analysis of variance test for group by sex interaction on the attitude variables. Groups formed by instructional methods and sex categories were compared on scores from attitude scales. The Ideas and Preferences In-

ventory provided scores measuring attitudes toward mathematics in general and the Geometry Attitude Survey, administered at the end of the experiment, measured attitudes toward geometry specifically.

Table 20. Analysis of Variance, group by sex interaction, based on attitude scores

Attitude Variables	df	F-ratio	Significance Level
Attitude toward mathematics	(1,199)	0.239	0.999
Attitude toward geometry	(1,199)	0.970	0.999

There is no indication from the results in Table 20 of an interaction between attitude scores and sex of student. The F-ratios are well below that required at the .05 level of significance. There is insufficient evidence to reject Hypothesis 11.

Summary of Findings

To summarize the data analyses the hypotheses will be grouped into three categories: (1) achievement comparisons, (2) attitude comparisons, and (3) method by pre-score interactions.

Achievement comparisons

Null Hypotheses 1, 2, and 3 refer to comparisons of the experimental and control groups on geometry achievement scores. Analysis of Covariance, with IQ as the covariate, was used to test for significant differences of group means.

Hypothesis 1: There is no significant difference between mean scores of the experimental and control groups when tested on achievement in traditional geometric properties by teacher-made unit examinations.

Hypothesis 2: There is no significant difference between mean scores of the experimental and control groups when given a standardized, full-year geometry achievement test.

Hypothesis 3: There is no significant difference in performance of the experimental and control groups, as measured by a teacher-made achievement test, when a coordinate geometry unit is studied that includes coordinate geometry proofs.

Table 21 summarizes the results of the data analyses used to test Hypotheses 1, 2, and 3. The five percent level of significance was used to test the null hypotheses. As Table 21 reveals (see page 85), null Hypotheses 1, 2, and 3 were not rejected at the .05 level of significance. The data collected from common questions on unit examinations and the ETS Cooperative Mathematics geometry test revealed no significant group differences for the total scales. Group means did test significantly different for a subset of six items on the ETS test which were categorized as "logic and nature of proof" items.

Table 21. Summary of Analysis of Covariance tests for difference between group means on achievement variables

Hypothesis	Variate	F-ratio	Final Decision for Hypothesis
1.	Sem. I teacher-made tests	.021	
1.	Full-year teacher-made tests (Coordinate Geom. excluded)	1.042	
1.	Decision		not rejected
2	Logic and proof items on ETS Cooperative Math. test	7.510**	
2	ETS Cooperative Math. test, logic and proof excluded	1.250	
2	Total ETS Cooperative Mathematics test	2.249	
2	Decision		not rejected
3.	Coordinate geometry proof subtest	.800	
3.	Coordinate geometry total test	1.482	
3.	Decision		not rejected

**Significant at $P < 0.01$.

Attitude comparisons

Null Hypotheses 4 and 5 refer to comparisons between the two treatment groups on scales measuring attitudes toward mathematics in general and toward geometry specifically.

Hypothesis 4: There is no significant difference between mean scores of the experimental and control groups when given an attitude-toward-mathematics scale.

Hypothesis 5: There is no significant difference between mean scores of the experimental and control groups when given an attitude-toward-geometry scale.

The Ideas and Preferences Inventory was administered at the beginning of the experiment as a pre-test and to serve as a base for measuring attitude changes during the experiment. T-tests for the various subscales of the attitude pre-test revealed no significant differences in group means. T-tests were again used to test for group differences in attitude changes at the end of each semester. Table 22 shows the results of these t-test comparisons and also comparisons using scores from the Geometry Attitude Survey which was administered at the end of the experiment.

Table 22 (see page 87) reveals that there were no significant differences between group means when full-year total scores were compared. Null Hypotheses 4 and 5 were not rejected. However, when one-semester attitude changes are considered, there are significant differences on the total scale and also on subscales Math vs. Non-Math, Fun vs. Dull, and Facilitating Anxiety. Experimental group means increased during Semester I on these subscales while control group means decreased on the same scales (see Table 13). During the first semester, fundamental geometry topics were being covered in both treatment groups. The major instructional difference was that in the control group a strong emphasis was placed on the development of formal two-column

Table 22. Summary of T-Tests for difference between group means on attitude variables

Hypothesis	Variate	Level of Significance			Final Decision for Hypothesis
		Sem. I	Sem. II	Full-year	
4	Math vs. Non-Math subscale	.05*(E)	.49	.39	
4	Fun vs. Dull subscale	.04*(E)	.37	.20	
4	Composite subscale	.56	.60	.90	
4	Easy vs. Hard subscale	.25	.31	.60	
4	Facilitating Anxiety subscale	.02*(E)	.00**(C)	.24	
4	Debilitating Anxiety subscale	.84	.11	.18	
4	Self-Concept subscale	.63	.25	.69	
4	Total scale	.05*(E)	.04*(C)	.82	
4	Decision				not rejected
5	Interest Pleasure subscale			.85	
5	Difficulty subscale			.58	
5	Relevance subscale			.64	
5	Total scale			.92	
5	Decision				not rejected

*Significance at $P < 0.05$, (E) indicates significance in favor of the experimental group and (C) indicates significance in favor of the control group.

**Significance at $P < 0.01$.

proofs as a means for logical justification of geometric properties, while in the experimental group, topics were developed in a less formal manner with any proof arguments being made in an informal paragraph style (orally in most cases). During the second semester, less emphasis was placed on proofs for the control group while a major thrust for the experimental group (9 weeks) was on applications of geometric properties (i.e., career units). Control group means increased during the second semester on the total scale and on the Facilitating Anxiety subscale while the experimental group means decreased on the same scales (see Table 13).

Method by pre-score interactions

Hypotheses 6 through 11 refer to possible interaction between the two instructional methods of this study and variables related to geometry success, and for which data were collected before the experiment began. The relationships between the treatment variable and IQ, sex, and mathematics grade-point average were examined using both achievement and attitude as criterion variables. The relationships between instructional method and IQ and instructional method and grade-point average were examined through comparison of Pearson product-moment correlation coefficients. Hypotheses 6 through 9 were tested by these comparisons.

Hypothesis 6: There is no significant difference in the relationship of achievement and IQ scores for the experimental and control groups.

Hypothesis 7: There is no significant difference in the relationship of geometry achievement and grades seven through nine mathematics grade-point averages for the experimental and control groups.

Hypothesis 8: There is no significant difference in the relationship of scores from attitude scales and IQ scores for the experimental and control groups.

Hypothesis 9: There is no significant difference in the relationship of scores from attitude scales and grades seven through nine mathematics grade-point averages for the experimental and control groups.

Correlation coefficients were computed for experimental and control groups relating total achievement scores to IQ and to mathematics grade-point average and relating total attitude scores to IQ and to mathematics grade-point average. When testing for treatment group differences, only one comparison indicated a significant difference. Coefficients of correlation between total scores on the Ideas and Preferences Inventory (attitudes toward mathematics) and IQ were .3222 for the experimental group and -.0103 for the control group; these differ at the .05 level of significance. This indicates that students at the same IQ levels in both treatment groups reacted differently to the two instructional methods. All other correlation coefficients were positive except the correlation between attitude-toward-mathematics scores and mathematics grade-point average in the experimental group. The two groups did not differ significantly

for this comparison however, and the negative correlation for the experimental group was rather weak ($-.0690$). Hypothesis 8 was rejected at the .05 level, Hypotheses 6, 7, and 9 were not rejected.

Hypotheses 10 and 11 refer to the interaction of instructional method and sex of the student.

Hypothesis 10: When achievement is used as the criterion variable, there is no significant interaction between instructional method and sex of the student.

Hypothesis 11: When attitude is used as the criterion variable, there is no significant interaction between instructional method and sex of the student.

These two hypotheses were tested by the analysis of variance procedure. No significant interactions were indicated by the tests made, so null Hypotheses 10 and 11 were not rejected.

CHAPTER V. SUMMARY, DISCUSSION, AND RECOMMENDATIONS

Summary of Study

The purpose of this study was to compare the effectiveness of formal and informal approaches to high school geometry instruction. The formal course was the traditional axiomatic development of geometry, considering carefully the properties of an abstract mathematical system, and using the formal two-column proof as a vehicle for instructional development of the content. The text, *Modern School Mathematics Geometry*, 1969 edition, by Jurgenson, Donnelly, and Dolciani was used. The informal course included basically the same geometric topics developed with little emphasis on the abstract nature of mathematics, informal proofs of geometric properties (usually oral), and a greater emphasis on practical applications. Text materials for informal geometry were prepared by the two participating instructors.

Participants in the study included 24 eleventh graders and 195 tenth graders who did not participate in the accelerated mathematics program available at the eighth grade level, but completed first year algebra in grade nine. A table of random numbers was used to divide the participants into experimental and control groups. The experimental group (102 students) studied informal geometry and the control group (117 students) studied formal geometry.

The criterion variables used to measure success in the

two geometry courses were achievement and attitude. Teacher-made unit examinations and the ETS cooperative Mathematics Tests: Geometry, Form A were used to compare the experimental and control groups on achievement. The Ideas and Preferences Inventory, Forms 111B and 117 (31), developed for the five-year National Longitudinal Study of Mathematical Abilities (1962-65) and the Geometry Attitude Survey, developed by Felix Labaki (16), were used for attitude comparisons. The Ideas and Preferences Inventory, measuring attitudes toward mathematics in general, was administered at the beginning of the experiment, at the end of Semester I, and near the end of Semester II. Scores from these scales were used to compare mean attitude changes for the experimental and control groups. The Geometry Attitude Survey, measuring attitudes toward geometry specifically, was administered at the end of the experiment.

The multiple regression method of analysis of covariance was used to analyze all achievement data. Achievement means for treatment groups were compared at the .05 level of significance after adjustment for the covariate IQ. T-tests were used to compare experimental and control group means on all attitude scales. The standard analysis of variance and covariance procedures were used to measure possible interaction between the categorical variables, instructional method and sex of student. To assess the relationship of instruc-

tional method to the metric variables, IQ and grade-point average, correlation coefficients were compared using the Fisher's Z_r transformation procedure (13, pp. 187-8).

The hypotheses tested in this study were as follows:

Hypothesis 1: There is no significant difference between mean scores of the experimental and control groups when tested on achievement in traditional geometric properties by teacher-made unit examinations.

Hypothesis 2: There is no significant difference between mean scores of the experimental and control groups when given a standardized, full-year geometry achievement test.

Hypothesis 3: There is no significant difference in performance of the experimental and control groups, as measured by a teacher-made achievement test, when a coordinate geometry unit is studied that includes coordinate geometry proofs.

Hypothesis 4: There is no significant difference between mean scores of the experimental and control groups when given an attitude-toward-mathematics scale.

Hypothesis 5: There is no significant difference between mean scores of the experimental and control groups when given an attitude-toward-geometry scale.

Hypothesis 6: There is no significant difference in the relationship of achievement and IQ scores for the experimental and control groups.

Hypothesis 7: There is no significant difference in the relationship of geometry achievement and grades seven through nine mathematics grade-point averages for the experimental and control groups.

Hypothesis 8: There is no significant difference in the relationship of scores from attitude scales and IQ scores for the experimental and control groups.

Hypothesis 9: There is no significant difference in the relationship of scores from attitude scales and grades seven through nine mathematics grade-point averages for the experimental and control groups.

Hypothesis 10: When achievement is used as the criterion variable, there is no significant interaction between instructional method and sex of the student.

Hypothesis 11: When attitude is used as the criterion variable, there is no significant interaction between instructional method and sex of the student.

Discussion of Findings

Only one of the eleven null hypotheses tested in this study was rejected; Hypothesis 8 was rejected at the .05 level of significance. Although failure to reject does not imply acceptance, the fact that the data collected in this study would not support rejection of ten of the hypotheses leads to certain implications about the comparative effectiveness of formal and informal high school geometry.

A major objective for both treatment groups was to gain knowledge of the geometric facts and relations that are useful in subsequent mathematics courses and/or practical applications. This content achievement was measured by teacher-made unit tests and by the standardized full-year ETS achievement test. Overall achievement, as measured by these tests, was not significantly different for the two treatment groups. This evidence supports the belief that the single-abstract-system approach (formal geometry) offers no significant advantage for learning a major portion of

traditional geometric content.

Formal two-column proofs played a major role in the axiomatic development of formal geometry. Informal geometry included basically the same geometric topics, but the content was developed through informal proof arguments (usually oral). Coordinate Geometry Methods and Proofs, the last unit of the experiment, was studied by both treatment groups and included coordinate proofs of geometry theorems. The performance on this unit was not significantly different for the experimental and control groups. Since coordinate proofs are more likely to be encountered in the future, by a majority of the students participating in this study (students in the accelerated program were not included), the data from this unit suggest that performance in subsequent mathematics courses will not be adversely affected by the absence of formal proofs in informal geometry. Additional research is needed to test this possibility.

The only indication of an achievement advantage by either treatment group came from the comparison of group means on a set of six items categorized as "logic and nature of proof" items on the ETS Cooperative Mathematics test. The mean of 2.69 for the experimental group and 3.24 for the control group tested significantly different at the .01 level. These six items related more to the material studied by formal geometry students and they performed better on them.

Although the implication here is that the formal geometry students possessed a better understanding of mathematical proofs, this did not seem to be an advantage when both treatment groups studied Coordinate Geometry Methods and Proofs, the last unit of the experiment.

The second major area of concern in this study was student attitudes: attitudes toward mathematics in general and toward their high school geometry course specifically. Attitudes toward mathematics were measured at the beginning of the school year, at the end of Semester I, and again near the end of Semester II. Experimental and control group means for these scales did not differ significantly at the beginning of the experiment and they did not differ significantly at the end of the experiment. However, significant differences did appear when one-semester attitude changes were compared. At the end of Semester I, attitude scores indicated a positive change for the experimental group and a negative change for the control group on three subscales: Math vs. Non-Math, Fun vs. Dull, and Facilitating Anxiety. The differences in group means on these subscales were significant at the .05 level. During the second semester, experimental group attitudes changed negatively while control group attitudes changed positively, so that when treatment group means were compared for total change, no significant differences were obtained. Since the major emphasis

on formal proofs came during Semester I in formal geometry, this is a possible explanation for the negative change in attitude during Semester I and a positive change during Semester II for the control group. The major emphasis on geometry applications (career units) in the informal geometry course came during Semester II. Informal geometry students showed an increase in attitudes during Semester I and a decrease during Semester II on all subscales. This seems to imply that the study of career units caused the negative change in attitudes, however, the students seemed to express favorable opinions to the instructors while these units were being studied. A number of research studies at the secondary level have indicated negative change in mathematics attitudes as the school year progresses, so it is possible the negative change was more related to the length of the school year than to the specific content being studied. It was encouraging to note that the overall yearly change in attitude toward mathematics was positive for both treatment groups when all attitude subscales were combined. When the Geometry Attitude Survey was administered at the end of the experiment, measuring the attitudes of the experimental and control group students toward their respective geometry courses, the group means did not differ significantly.

Statistical tests were also made to locate possible

interactions between the informal and formal approaches to geometry and the independent variables: IQ, previous mathematics grade-point average, and sex of student. Both achievement and attitude were used as criterion variables in these tests. Only one of these tests indicated a significant difference; the coefficients of correlation between the total score on the attitude toward mathematics scale and IQ were significantly different for the two treatment groups. The correlation between attitude and IQ for the experimental group was .3222 and for the control group it was -.0103. In the investigator's opinion, a possible explanation is that the study of proofs and emphasis on the abstract organization of geometry for the control group caused attitudes to be less predictable. Many otherwise good mathematics students expressed a dislike for the extended emphasis on formal proofs.

After examining all evidence, it appears that the informal approach to geometry (experimental) is a valid alternative to the traditional axiomatic development of the subject. In this experiment there has been no indication of a significant advantage for either instructional approach as measured by total scores for attitudes or knowledge of geometric facts and relations. This does not mean that there are no advantages of one method over the other. There are worthwhile instructional objectives included in both geom-

etry courses that have not been measured in this study. An understanding of the structure of an abstract mathematical system is a worthwhile objective for some students; the control group received more emphasis on this topic. Application of geometric properties is a worthwhile instructional objective; the experimental group spent more time on application problems. These variables (and others) were not measured thoroughly in this study.

Historically, the careful axiomatic development of high school geometry has been the chief vehicle for teaching the abstract nature of mathematics. During the last two decades, greater emphasis has been placed on the formal structure of mathematics at every level. This has led some educators to question the value of a rigorous axiomatic development of geometry. The results of analyses of data in this study suggest that emphasis on the structure of geometry offers no significant advantage for achievement in learned geometric facts and relations or for developing favorable attitudes toward mathematics. More research is needed on this issue but if many of the accepted objectives of high school geometry can be reached without the axiomatic development, then content and instructional methods would become more flexible and the geometry course could be more easily adapted to a variety of student needs. It is this investigator's opinion that the single-abstract-system objective

should still be a primary objective for many students, but alternatives such as the one used in this study seem to be appropriate for a number of tenth grade mathematics students.

Limitations

As with most research, there are limitations affecting the applicability of the results of this study.

First, the characteristics of the subjects participating in the experiment must be taken into consideration. Most participants were tenth graders who had completed first year algebra in the ninth grade. Although the opportunity was available for capable students, the students in this study did not accelerate in their mathematics sequence at the eighth grade level. Hence, many of the higher ability mathematics students were not included in this study. The average IQ for 219 participants was approximately 113 with IQ's ranging from 85 to 135.

The course content and text materials must also be considered before utilizing the results of this study. The text for the experimental group was prepared by the participating instructors and is not commercially available (see Chapter III for description).

Implications from the results of this study are limited to the variables measured by the testing instruments employed. Several variables associated with the informal and

formal geometries of this study were not measured. The results are not sufficiently inclusive to recommend one instructional method over the other.

Caution must be exercised when the results of this study are generalized to a different educational setting. Variables are present in each school system that affect student needs and their attitudes toward educational experiences.

Recommendations for Further Research

Further experimental research is needed to focus on the following question: Is an axiomatic approach which focuses upon the logical structure of high school geometry and uses deductive proof as the primary method of verification superior to other teaching methods? Many claims have been made on both sides of this question but most have been theoretical arguments.

Additional research is needed to determine the effects of formal and informal geometry instruction, as described in this study, on variables related to logical reasoning and applications of geometric properties.

Additional research is needed to determine the effects of formal and informal geometry instruction, as described in this study, on performance in subsequent mathematics courses.

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IDEAS AND PREFERENCES INVENTORY

INSTRUCTIONS: This is not a test. There are no "right" or "wrong" answers to any of the questions. Just answer them as honestly as you can.

The questions ask you to tell how you feel about many different things. Your answer to each question should tell how you feel about it.

Some questions ask about experiences you have had in the past. When you answer these, think back to the experiences you have had in the last year or so.

Please work carefully but quickly. Do not spend a long time on any one question. Just fill in the answer that seems best to you at the moment. Please answer all the items, and give only one answer for each item.

Some questions have a blank space in the middle. Four ways to fill the blank space are given beneath each sentence. Look at sample question 0.

0. I like summer _____ than winter.

- A. a lot more
- B. a little more
- C. a little less
- D. a lot less

Which one of the four ways tells best how you like summer as compared with winter: A, or B, or C, or D? Mark your answer on the separate answer sheet for sample question 0.

For other questions you are just to tell how you feel about each statement by selecting one of the ways given beneath the statement. Look at sample question 00.

00. It is more fun to play outdoors in winter than in summer.

- G. strongly agree
- H. agree
- J. disagree
- K. strongly disagree

Which one of the four ways tells best how you feel about the statement: G, or H, or J, or K? Mark your answer on the separate answer sheet for sample question 00.

If you have any questions while you are working, please raise your hand. Wait for the signal to begin.

1. I like story books _____ than mathematics books.
 - A. a lot more
 - B. a little more
 - C. a little less
 - D. a lot less

2. I like the problem " $359 - 574 + 6840 - 999 - 46937 + 9748 + 97483 = ?$ " _____ than the problem "Jane is half as tall as Dick. Joe is half as tall as Jane. Mark is half as tall as Joe. Dick is 60 inches tall. How tall is Joe?"
 - G. a lot more
 - H. a little more
 - J. a little less
 - K. a lot less

3. I like doing mathematics _____ than doing anything else.
 - A. a lot more
 - B. a little more
 - C. a little less
 - D. a lot less

4. I like writing answers to social studies questions _____ than doing word problems in mathematics.
 - G. a lot more
 - H. a little more
 - J. a little less
 - K. a lot less

5. I like mathematics books _____ than social studies books.
 - A. a lot more
 - B. a little more
 - C. a little less
 - D. a lot less

6. I like subtracting fractions _____ than reading a story about Brazil.
 - G. a lot more
 - H. a little more
 - J. a little less
 - K. a lot less

7. I would like to teach English _____ than I would like to teach mathematics.
- A. a lot more
 - B. a little more
 - C. a little less
 - D. a lot less
8. The subject I enjoy least is mathematics.
- G. strongly agree
 - H. agree
 - J. don't know
 - K. disagree
 - L. strongly disagree
9. For most jobs it is more important to be well rounded and broadly educated than to know mathematics.
- A. strongly agree
 - B. agree
 - C. don't know
 - D. disagree
 - E. strongly disagree
10. I cannot understand how some students think mathematics is fun.
- G. strongly agree
 - H. agree
 - J. mildly agree
 - K. mildly disagree
 - L. disagree
 - M. strongly disagree
11. Mathematics is boring.
- A. strongly agree
 - B. agree
 - C. don't know
 - D. disagree
 - E. strongly disagree
12. Mathematics is fun.
- G. strongly agree
 - H. agree
 - J. don't know
 - K. disagree
 - L. strongly disagree

13. My parents think mathematics is not very practical.
- A. strongly agree
 - B. agree
 - C. don't know
 - D. disagree
 - E. strongly disagree
14. No matter how hard I try, I cannot understand mathematics.
- G. strongly agree
 - H. agree
 - J. don't know
 - K. disagree
 - L. strongly disagree
15. Mathematics is a subject which is more difficult to understand than any other subject.
- A. strongly agree
 - B. agree
 - C. don't know
 - D. disagree
 - E. strongly disagree
16. Most mathematics is too concerned with ideas to be really useful.
- G. strongly agree
 - H. agree
 - J. don't know
 - K. disagree
 - L. strongly disagree
17. Except for those who are going to be scientists or engineers, most students would rather take other courses than mathematics.
- A. strongly agree
 - B. agree
 - C. don't know
 - D. disagree
 - E. strongly disagree
18. My parents think mathematics is my most important subject.
- G. strongly agree
 - H. agree
 - J. don't know
 - K. disagree
 - L. strongly disagree

19. There is so much hard work in mathematics that it takes the fun out of it.
- A. strongly agree
 - B. agree
 - C. don't know
 - D. disagree
 - E. strongly disagree
20. I would like mathematics better if it were not made so hard in class.
- G. strongly agree
 - H. agree
 - J. don't know
 - K. disagree
 - L. strongly disagree
21. I can get along perfectly well in everyday life without mathematics.
- A. strongly agree
 - B. agree
 - C. don't know
 - D. disagree
 - E. strongly disagree
22. Mathematics is easier for me than my other subjects.
- G. strongly agree
 - H. agree
 - J. don't know
 - K. disagree
 - L. strongly disagree
23. Mathematics is so hard to understand that I do not like it as well as other subjects.
- A. strongly agree
 - B. agree
 - C. don't know
 - D. disagree
 - E. strongly disagree
24. To do well in mathematics, you have to be smarter than you have to be to do well in reading.
- G. strongly agree
 - H. agree
 - J. don't know
 - K. disagree
 - L. strongly disagree

25. Most students work very hard to do well in mathematics.
- A. strongly agree
 - B. agree
 - C. don't know
 - D. disagree
 - E. strongly disagree
26. Mathematics is more of a game than it is hard work.
- G. strongly agree
 - H. agree
 - J. don't know
 - K. disagree
 - L. strongly disagree
27. Nervousness while taking a mathematics test keeps me from doing well.
- A. always
 - B. often
 - C. sometimes
 - D. rarely
 - E. never
28. I work best on mathematics tests that are important.
- G. always
 - H. usually
 - J. sometimes
 - K. hardly ever
 - L. never
29. When I have been doing poorly in mathematics, my fear of a bad grade keeps me from doing my best.
- A. never
 - B. hardly ever
 - C. sometimes
 - D. usually
 - E. always
30. I keep my mathematics grades up mainly by doing well on the big tests rather than on homework and quizzes.
- G. always
 - H. usually
 - J. sometimes
 - K. hardly ever
 - L. never

31. When I am poorly prepared for a mathematics test, I get upset and do even less well than I expected.
- A. never
 - B. hardly ever
 - C. sometimes
 - D. usually
 - E. always
32. The more important the mathematics test, the less well I seem to do.
- G. always
 - H. usually
 - J. sometimes
 - K. hardly ever
 - L. never
33. Whether or not I am nervous before taking a mathematics test, once I start I seem to forget my nervousness.
- A. I always forget my nervousness
 - B. I usually forget my nervousness
 - C. I sometimes forget my nervousness
 - D. I rarely forget my nervousness
 - E. I never forget my nervousness
34. During mathematics tests I find I cannot answer questions even though I usually know the answers and might remember them when the test is over.
- G. always
 - H. Often
 - J. sometimes
 - K. hardly ever
 - L. never
35. Nervousness while taking a mathematics test helps me to do better.
- A. it never helps
 - B. it usually doesn't help
 - C. it sometimes helps
 - D. it usually helps
 - E. it always helps
36. When I start a mathematics test, I find it easy to concentrate.
- G. always
 - H. often
 - J. sometimes
 - K. hardly ever
 - L. never

37. I find that my mind goes blank at the beginning of a mathematics test and it takes me a few minutes before I can answer the questions.
- A. I almost always blank out at first
 - B. I usually blank out at first
 - C. I sometimes blank out at first
 - D. I hardly ever blank out at first
 - E. I never blank out at first
38. I look forward to mathematics tests.
- G. never
 - H. hardly ever
 - J. sometimes
 - K. usually
 - L. always
39. I get so tired from worrying about a mathematics test that I find I almost don't care how well I do by the time I start it.
- A. I never feel this way
 - B. I hardly ever feel this way
 - C. I sometimes feel this way
 - D. I often feel this way
 - E. I almost always feel this way
40. Because I worry so much about not being able to finish mathematics tests in the required time, I always do worse than the rest of the class.
- G. always
 - H. usually
 - J. sometimes
 - K. seldom
 - L. never
41. Although last minute studying before mathematics tests does not work for most people, I find that I can learn material just before a test and remember it for use on the test.
- A. always
 - B. usually
 - C. sometimes
 - D. seldom
 - E. never

42. I enjoy taking a hard mathematics test more than an easy one.
- G. always
 - H. often
 - J. sometimes
 - K. hardly ever
 - L. never
43. I find myself reading mathematics test questions without understanding them, and I must go back over them so they will make sense.
- A. never
 - B. rarely
 - C. sometimes
 - D. often
 - E. almost always
44. The more important the mathematics test, the better I seem to do.
- G. always
 - H. usually
 - J. sometimes
 - K. rarely
 - L. never
45. When I don't do well on difficult questions at the beginning of a mathematics test, it tends to upset me so that I don't do well even on the easy questions later on.
- A. never
 - B. rarely
 - C. sometimes
 - D. usually
 - E. always
46. I find it hard to talk in front of my mathematics class.
- G. strongly agree
 - H. agree
 - J. mildly agree
 - K. mildly disagree
 - L. disagree
 - M. strongly disagree

47. I am very proud of my mathematics school work.
- A. strongly agree
 - B. agree
 - C. mildly agree
 - D. mildly disagree
 - E. disagree
 - F. strongly disagree
48. I try to do the very best work in mathematics that I can.
- G. strongly agree
 - H. agree
 - J. mildly agree
 - K. mildly disagree
 - L. disagree
 - M. strongly disagree
49. I like to be called on in mathematics class.
- A. strongly agree
 - B. agree
 - C. mildly agree
 - D. mildly disagree
 - E. disagree
 - F. strongly disagree
50. I think I am not doing very well in mathematics class.
- G. strongly agree
 - H. agree
 - J. mildly agree
 - K. mildly disagree
 - L. disagree
 - M. strongly disagree